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MARKOV SWITCHING MEAN VECTOR AUTOREGRESSIVE (MSM VAR) MODELLING OF INFLATION RATE AND CRUDE OIL PRICE INTERDEPENDENCE IN NIGERIA

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Abstract: This study used the Markov Switching Mean Vector Autoregressive (MSM VAR) Models to model the interdependence between Nigeria's inflation rate and crude oil prices. Monthly data from January 2006 to December 2019 were gathered from the Central Bank of Nigeria Statistical Bulletin for the study. The upward and downward movement in the series revealed by the time plot suggested that the series exhibited a regime-switching pattern: the period of expansion and contraction. The Augmented Dickey-Fuller test was used to screen for stationary and the variables were stationary at first differences. The information criteria were used to test the number of regimes and 2 regimes were selected. Eight MSM-VAR models were estimated. The best model chosen based on the least information criterion was the Markov-switching mean heteroscedasticity – vector Autoregressive (MSMH-VAR) model with AIC (8.597689) and SC (8.944167). The model was used to predict the series' values over a one-year cycle (12 months).

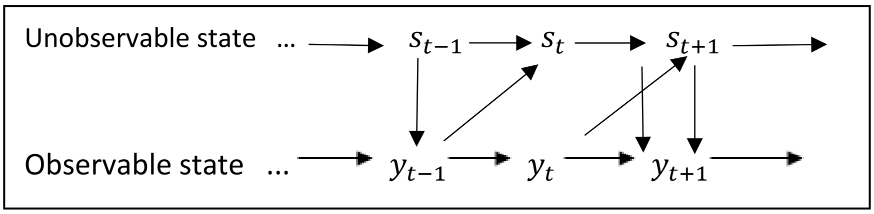
Keywords: inflation rate, Crude oil prices, Markov Switching Mean Heteroscedasticity, VAR, forecast

1.0 Introduction

For a long time, the price of crude oil and the rate of inflation in Nigeria have been inextricably linked. As the price of crude oil rises, the rate of inflation rises with it. Oil is a major source of income in Nigeria's economy, and it is used in vital activities such as fuelling, transportation, and home heating, among others.[4] There is a progression of rises and falling (expansions and contractions) between the two series, which fluctuates around a higher and lower. It would be unreasonable to expect a single linear or multivariate model to capture these distinct behaviours for such data especially the duration of stay in a regime and the transmission from one regime to another.

Hamilton [9] introduced the Markov switching model, also known as the regime-switching model, which is one of the most commonly used nonlinear time series models. This model can capture more complex dynamic patterns by allowing switching between each regime. The switching mechanism of the Markov Switching Model is regulated by observable and unobservable state variables that follow a first-order Markov chain.

Figure 1.1 Switching Process



[15]
Unobservable state s_t, *Observable state* y_t;

The Markovian property, in particular, allows the current value of the state variable to be influenced by its recent past values, therefore, a structure may be dominant for a random time before being replaced by another structure when switching occurs. The regime-switching model differs from [16] random switching model, in which the switching events are time-independent. As a result, the Markov switching model is well suited in modelling the nonlinear time series model that explains the complex patterns over time (expansion and contraction).

2.0 Literature Reviews

[9] examined the parameter of an autoregressive model as a discrete state Markov switching model. He tested the growth rate of the non-stationary series of the Postwar in the US and read GNP using probabilities inferences about whether and when they may have occurred based on the observed behaviour of the series. He conclusion, the series fluctuates periodical from a positive growth rate to a negative growth rate

According to [11], the MS-VAR framework constitutes the multivariate generalization of Hamilton's single equation model. In these extended models, there is an unobserved state driven by an ergodic Markov process that is common to all series. In a sequence of papers, Krolzig has studied the statistical analysis of the Markov Switching Vector Autoregressive (MS-VAR) models and their application to dynamic multivariate systems. [6] discussed the characterization and the testing of business cycle asymmetries based on MS-VAR models.

[2] studies the average growth rate of gross domestic product from (1951-1984) in terms of the recession period. He used Markov switching regression models. The result revealed the presence of recession is better than expansion and the model adopted is MS-AR(2).

[7] examined the currency crises in Asia. He used the Markov switching vector autoregressive model to model the series from (1980-199). Three approaches were adopted: The probability of a currency crisis, the probability of a turbulent regime and the expected value of the index speculative pressure. The studies showed that MS-VAR models with time-varying transitional probability are a good process to use in building an early warning system of the currency period.

[3] studies American industrial production function using the technique of Hamilton and Krolzig to compare the MSVARLIB (Gauss library) and the Krolzig MS-VAR for (ox). He concludes that the MSVARLIB for the Gauss library is better than the ox package by Krolzig.

[14] examined the impact of oil price on the South African GDP growth rate using the Bayesian Markov switching vector autoregressive model. They are found out that the oil prices in South African fluctuate around a period of low and high growth (low regime and high regime). The accepted model for predicting the read output of growth rate in the region is the low growth rate regime which is shorter compared to the high regime growth.

[13] examined the best approach in financial series by comparing the vector autoregressive models, Markov switching models and Bayesian inference for Markov switching vector autoregressive models with existing algorithms using monte-Carlo (SMC-estimators). However, none of these studies explicitly analysed the stochastic properties of the Nigerian economic structure.

3.0 Methodology

3.1 Time Plot

When dealing with time-series data, the first step in the study is typically to create a time plot of the data to analyze a simple descriptive measure of the series' key properties. The graph of inflation and crude oil prices are plotted against time to give us a sense of the overall movement of the original data over time, as well as whether the pattern is steady or fades over time. This graph also shows the researcher the following: Trend (upward or downward) movement over time, Seasonal fluctuation, Constant variances, and the duration of expansion and contraction.

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3.2 Markov Switching Model

In certain cases, the regime in effect at any given time may be directly observed. In general, the regime is unobserved, so the researcher must conclude which regime it is. Markov switching models are time series models in which variables can switch from one state to the next in a defined number of regimes. As part of the model, a stochastic mechanism that allows variables to switch between regimes according to an unobserved Markov chain was used to produce the regime shifts in the past and present. Threshold models and Markov-switching models are the two types of regime-switching models. The primary difference between these approaches is how the state process is modelled. Threshold models, introduced by [17], assume that regime shifts are triggered by the level of observed variables about an unobserved threshold. Markov-switching models, introduced by [9], assume that the regime shifts according to a Markov chain.

Markov-switching models also assume that S_t is the unobserved variable and y_t an observed variable. In contrast to threshold models, however, S_t is assumed to follow a particular stochastic process, namely an N state Markov chain. The development of Markov chains is described by their transition probabilities, given by:

$$\Pr(S_t = i/S_{t-1} = j, S_{t-2} = q \ldots\ldots) = \Pr(S_t = i/S_{t-1} = j) = \text{Pij} \quad (3.1)$$

Where conditional on a value of j, we assume $\sum_{i=1}^{n} Pr_{ij}=1$. That is, the process specifies a complete probability distribution for S_t. In general, the Markov process allows regimes to be switched from one state to another and for regimes to be switched, more than once restrictions can be placed on Pr_{ij} to restrict the order of regime shifts. [10]

3.3 Markov Switching Vector Autoregressive Model

The MS-VAR models are used to predict VAR models when the regime shifts from an observable state to an unobservable state. This model is a general class of models that describe a non-linear data generation process as piecewise linear by constraining the process to be linear in each regime. The stochastic mechanism that produces the regime is assumed differently in these models. Markov-switching vector autoregressive can be considered as generalizations of the basic finite order VAR model of order p. If the process is subject to shifts in the regime, the stable vector autoregressive model with its time-invariant parameters might be inappropriate. The idea behind this class of models is that the parameters of the underlying data generating a process of the observed time series vector y_i depend upon the unobservable regime variable S_t which represents the probability of being in a different state at a different time following a first-order Markov chain. The description of the data-generating process is not complete by the observational equations. A model for the regime generating process has to be formulated which then allows deducing the evolution of regimes from the data.[6] The general characteristic of the Markov switching model is that the unobservable realization of the regime $S_t \epsilon (1,2, \ldots\ldots N)$ is governed by

(i) discrete-time,

(ii) discrete-state Markov stochastic process, which is defined by the transition probabilities

$$P_{ij}=\Pr(S_{t-j} = j \ / S_t=\text{i}). \quad \sum_{j=1}^{N} P_{ij}=1, \forall_{ij} \ \epsilon \{1,2, \ldots\ldots.N\} \quad (3.2)$$

(iii) In general, S_t follows an irreducible ergodic N state Markov process with the transition matrix

$$P=\begin{bmatrix} p_{11} & p_{12} & . & . & p_{1N} \\ p_{21} & p_{22} & . & . & p_{2N} \\ . & . & . & . & . \\ . & . & . & . & . \\ p_{i1} & p_{i2} & . & . & p_{iN} \end{bmatrix} \quad (3.3)$$

$$P_{iN}= 1\text{-}p_{11} - - - - - - - - - - - - - p_{iN-1} \quad (3.4)$$

$$for\ i = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots N$$

The assumptions of ergodicity and irreducibility are important for the theoretical properties of Markov switching vector autoregressive models.

3.4 Markov Switching Vector Autoregressive Model (MS(N)-VAR(P))

The general specification of an MS(m)-VAR(p) model, all parameters of the autoregressive are conditioned on the state $\boldsymbol{S_t}$ of the Markov chain such that each regime (m)-VAR(p) parameters as following as regime proces. Generally [12]

$$Y_t = v_{s_t} + \sum_{i=1}^{p} b_{i,s_t} Y_{t-i} + \boldsymbol{\sigma}_{s_t}^{1/2} \boldsymbol{\varepsilon}_i \quad (3.5)$$

$$Y_t = \begin{cases} v_1 + b_{1,1} y_{t-1} + \cdots \ldots \ldots \ldots + b_{p,1} y_{t-p} + \sigma_1^{1/2} \varepsilon_i \text{ if } s_t = 1 \\ \vdots \\ v_m + b_{1,m} y_{t-1} + \cdots \ldots \ldots \ldots + b_{p}, m y_{t-p} + \sigma_m^{1/2} \varepsilon_i \text{ if } s_t = \text{m} \end{cases} \quad (3.6)$$

Where $Y_t = \begin{pmatrix} Y_1 \\ \vdots \\ Y_p \end{pmatrix}$, $b_i = \begin{pmatrix} b_{11} & \cdots & b_{1p} \\ \vdots & & \vdots \\ b_{p1} & \cdots & b_{pp} \end{pmatrix}$, $Y_{t-i} = \begin{pmatrix} Y_{t-1} \\ \vdots \\ Y_{t-p} \end{pmatrix}$, $s_t = \begin{cases} 1 \text{ if } s_t = 1 \\ \vdots \\ m \text{ if } s_t = m \end{cases}$

$v_{s_t} = \begin{pmatrix} v_{1,s_t} \\ \vdots \\ v_{1,s_t} \end{pmatrix}$, $\boldsymbol{\sigma}_{s_t}^{1/2}$ is the choleski decomposition of the return shock covariance $\varepsilon_i \sim N(0, \boldsymbol{\sigma}_{s_t}^{1/2})$

For analytical applications, however, it may be more useful to use a model in which only certain parameters are conditioned on the state of the Markov chain and the rest are regime invariant. When the autoregressive parameters, the mean and intercepts, are regime dependent, and the error term is heteroscedastic or homoscedastic, special MSVAR models may be implemented.

The MS(m)-VAR(p) models also allow for a variety of specifications. [11]

Established a common notation for expressing the models in which various parameters are subject to shifts regime with the varying and invariant states.

Note: μ=mean,v = intercept term, σ = Variance and b_i =matrix of autoregressive parameter

An overview of Markov switching vector autoregressive models is given in table 1.3. In many situations MSI(m)-VAR(p) and MSM`(m)–VAR(p) models will be sufficient, a regime-dependent covariance structure of the process might be considered as an additional feature.

3.5 Markov Switching Mean Vector Autoregressive Model (MSM(N)-VAR(P))

In a generalization of the mean-adjusted VAR(p) model. we would like to consider Markov switching mean vector autoregressive models of order (p) and (M) regimes as follow,

$Y_t - \varepsilon(S_t) = b_1(S_t)(y_{t-1} - \varepsilon(S_{t-1})) + \ldots + b_p(S_t)(y_{t-p} - \varepsilon(S_{t-m})) + \sigma_m^{1/2} \varepsilon_i$ (3.7) Where $\varepsilon_i \sim NID\ (0, \sigma(S_t))$, are shift functions describing the dependence of the parameter ϵ, $b_1 \ldots\ldots\ldots\ldots b_p$,$\sigma$ on the realized regime and $\varepsilon(S_{t-1})$ *is the* mean at difference regime[11]

$$\varepsilon_i(S_t) = \begin{cases} \varepsilon_1 \text{ if } S_t = 1 \\ \vdots \\ \varepsilon_m \text{ if } S_t = m \end{cases} \quad (3.8)$$

We define the general Markov switching models, the regime-dependent parameter as follows, to create a unique notation for each model:

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MSM-VAR	=	Markov-switching Mean –VAR
MSMA-VAR	=	Markov-switching Mean Autoregressive-VAR
MSMH-VAR	=	Markov-Switching Mean Heteroscedasticity –VAR
MSMAH-VAR	=	Markov-Switching Means Autoregressive heteroscedasticity –VAR
LINEAR-MVAR	=	Linear - Means VAR
MSH-MVAR	=	Markov-Switching Heteroscedasticity – Mean VAR
MSA-MVAR	=	Markov-switching Autoregressive – Mean VAR
MSAH-MVAR	=	Markov-Switching Autoregressive Heteroscedasticity – Mean VAR

Table 3.1: Models Specification of (MSM(M)-VAR(P))

Parameter/ Model	μ	b_i	σ
MSM-VAR	Varying	Invariant	Invariant
MSMH-VAR	Varying	Invariant	Invariant
MSMA-VAR	Varying		Invariant
MSMAH-VAR	Varying	Varying	Varying
LINEAR-MVAR	Invariant	Invariant	Invariant
MSA-MVAR	Invariant		Varying
MSH-MVAR	Invariant	Varying	Invariant
MSAH-MVAR	Invariant	Varying	Varying

Note: μ=*mean*, σ = *Variance and* b_i =*matrix of autoregressive parameter*

4.2 Representation of MSMH(2)-VAR(2) of Crude Oil Price and Inflation Rate

$$\begin{bmatrix} inf_t & -\varepsilon\,(S_t) \\ cop_t & -\varepsilon\,(S_t) \end{bmatrix} = \begin{pmatrix} b_0 \\ b_1 \end{pmatrix} + \begin{pmatrix} b_{1111} & b_{1112} \\ b_{1121} & b_{1122} \end{pmatrix} \begin{pmatrix} inf_{t-1} - \varepsilon\,(S_{t-1}) \\ cop_{t-1} - \varepsilon\,(S_{t-1}) \end{pmatrix} + \begin{pmatrix} b_{1211} & b_{1212} \\ b_{1221} & b_{1222} \end{pmatrix} \begin{pmatrix} inf_{t-2} - \varepsilon\,(S_{t-2}) \\ cop_{t-2} - \varepsilon\,(S_{t-2}) \end{pmatrix} + \begin{pmatrix} \sigma_{111}^{1/2} & \sigma_{112}^{1/2} \\ \sigma_{121}^{1/2} & \sigma_{122}^{1/2} \end{pmatrix} \begin{pmatrix} \varepsilon_{expan}^{inf} \\ \varepsilon_{expan}^{cop} \end{pmatrix} \quad \textbf{Regime 1}$$

$$\begin{bmatrix} inf_t & -\varepsilon\,(S_t) \\ cop_t & -\varepsilon\,(S_t) \end{bmatrix} = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} + \begin{pmatrix} a_{2111} & a_{2112} \\ a_{2121} & b_{2122} \end{pmatrix} \begin{pmatrix} inf_{t-1} - \varepsilon\,(S_{t-1}) \\ cop_{t-1} - \varepsilon\,(S_{t-1}) \end{pmatrix} + \begin{pmatrix} a_{2211} & a_{2212} \\ a_{2221} & a_{2222} \end{pmatrix} \begin{pmatrix} inf_{t-2} - \varepsilon\,(S_{t-2}) \\ cop_{t-2} - \varepsilon\,(S_{t-2}) \end{pmatrix} + \begin{pmatrix} \sigma_{211}^{1/2} & \sigma_{212}^{1/2} \\ \sigma_{221}^{1/2} & \sigma_{222}^{1/2} \end{pmatrix} \begin{pmatrix} \varepsilon_{expan}^{inf} \\ \varepsilon_{expan}^{cop} \end{pmatrix} \quad \textbf{Regime 2} \qquad \textbf{(4.1)}$$

Order of numbering the coefficients; Regime-lag-model-variable

3.6 Testing for the Number of Regimes

[8] Propose that in practices, it is advisable to use information criteria to test for the number of a regime that using the likelihood ratio test. Base on the above assumption by Guidolin the following information criteria are used to estimate the number of regimes between the series. The commonly used information criteria are:

(i) Akaike information criterion (AIC),
(ii) Hannah-Quinn information criterion (HQ)
(iii) Schwarz information criterion (SIC).

- AIC = $\ln|\Sigma_r| + \frac{2}{T}MK^2$ (3.9)

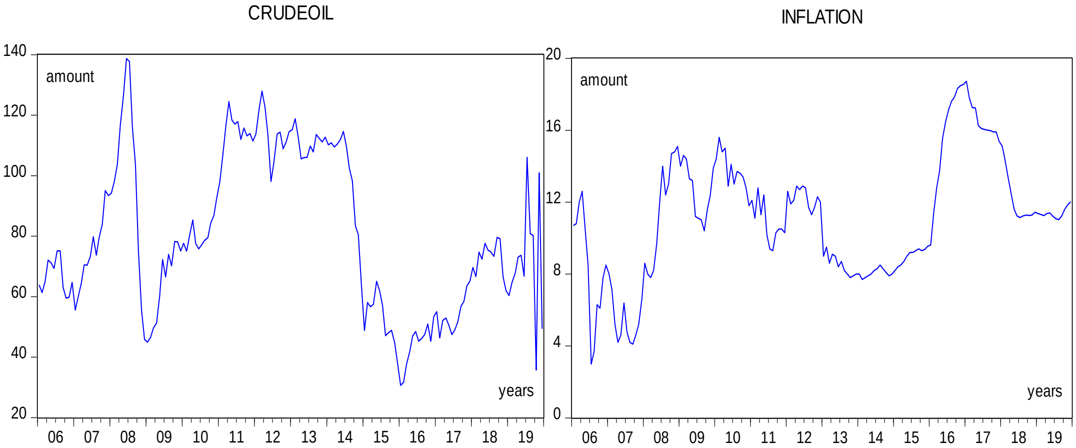
- HQ= $\ln|\Sigma_r| + \frac{2\ln T}{T} MK^2$ (3.10)
- SC = $\ln|\Sigma_r| + \frac{\ln T}{T} MK^2$ (3.11)

T is the number of observations (after accounting for lags)

M is the number of parameters estimated in each equation of the unrestricted system, including the constant. $\ln|\Sigma_r|$ is the natural log of the determinant of the covariance matrix of residuals of the restricted system. In each case, MK^2 is the number of VAR parameters in a model with order M. [5]

4.0 Result

4.1 Time Plot of Crude Oil Prices and Inflation Rate



A closer analysis of the time plots showed upward and downward movement in both series, suggesting that the series have a regime-switching pattern (a cycle of expansion and contraction in their movement), indicating a period of two regimes in the variable under study. The variables involved in this study were tested for stationarity using The Augmented Dickey-Fuller (ADF).at first differences, constant, linear trend and all the variables, the result showed the presence of unit root.

MSM: $Y_t - \mu_t(st) = \psi_1(s_t)(Y_{t-1} - \mu_t(s_{t-1}) + \cdots + \psi_p(s_t)(Y_{t-p} - \mu_t(s_{t-p})\ \varepsilon_t$ (3.39)

Table 4.1: Testing for the Number of Regimes

Since S_t= 1, 2, 3 …….. N, $N \geq 2$

Regime	AIC	SC
2	**9.441754**	**9.76176**
3	**9.67859**	**10.2345**
4	**10.2893**	**10.93452**

The best regime is selected based on minimum information criteria. The commonly used information criteria are the Akaike information criterion and Schwarz information criterion (SIC). Regime 2 has the minimum information criteria with an AIC of 9.441754 and SC of 9.76176.

Table 4.2: Selection of the Markov Switching mean Vector Autoregressive models

Estimated Models		Criterion	Schwarz criterion
1	MSM(2)-VAR(2)	9.441754	9.761760
2	**MSMH(2)-VAR(2)**	**8.597689**	**8.944167**
3	MSMAH(2)-VAR(2)	8.925824	9.45289
4	MSMA(2)-VAR(2)	9.155407	9.626005

5	LINEAR-MVAR(2)	9.36549	9.66676
6	MSH(2)-MVAR(2)	8.99343	8.951089
7	MSAH(2)-MVAR(2)	9.093009	9.601255
8	MSA(2)-MVAR(2)	9.124065	9.575839

The Markov switching vector autoregressive model allows for a great variety of specifications such as Markov-switching mean, Markov-switching autoregressive and Markov-switching heteroscedasticity. Eight models were estimated for Markov switching Mean- vector autoregressive models of crude oil prices and inflation rate. All variables were stationary at lag 1. The best model was selected based on the minimum information criterion. The MSMH(2)-VAR(2) is the best model with the following information criterion AIC (8.597689) and SC (8.944167).

4.2 Representation of MSMH(2)-VAR(2) of Crude Oil Price and Inflation Rate

$$\begin{bmatrix} inf_t & -\varepsilon(S_t) \\ cop_t & -\varepsilon(S_t) \end{bmatrix} = \begin{pmatrix} 0.078225 \\ 1.126303 \end{pmatrix} + \begin{pmatrix} 0.32319 & -0.191735 \\ -0.000436 & 0.28562 \end{pmatrix} \begin{pmatrix} inf_{t-1} - 0.0506 \\ cop_{t-1} - 0.07803 \end{pmatrix} + \begin{pmatrix} 0.112679 & 0.285623 \\ 0.02076 & 0.082813 \end{pmatrix} \begin{pmatrix} inf_{t-2} - 0.05081 \\ cop_{t-2} - 0.07766 \end{pmatrix} + \begin{pmatrix} 0.017286 & 0.033998 \\ 0.033998 & 28.56511 \end{pmatrix} \begin{pmatrix} \varepsilon^{inf}_{expan} \\ \varepsilon^{cop}_{expan} \end{pmatrix} \quad \textbf{Regime 1}$$

$$\begin{bmatrix} inf_t & -\varepsilon(S_t) \\ cop_t & -\varepsilon(S_t) \end{bmatrix} = \begin{pmatrix} -0.07606 \\ 0.550128 \end{pmatrix} + \begin{pmatrix} 0.32319 & -0.191735 \\ -0.000436 & 0.28562 \end{pmatrix} \begin{pmatrix} inf_{t-1} - 0.0506 \\ cop_{t-1} - 0.07803 \end{pmatrix} + \begin{pmatrix} 0.112679 & 0.285623 \\ 0.02076 & 0.082813 \end{pmatrix} \begin{pmatrix} inf_{t-2} - 0.05081 \\ cop_{t-2} - 0.07766 \end{pmatrix} + \begin{pmatrix} 1.552115 & -0.385852 \\ -0.38587 & 42.10228 \end{pmatrix} \begin{pmatrix} \varepsilon^{inf}_{expan} \\ \varepsilon^{cop}_{expan} \end{pmatrix} \quad \textbf{Regime 2}$$

(4.1)

4.3 The Transition Matrix of Crude Oil Prices and Inflation Rate

$$P_{ij} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} 2.71745 & -1.71745 \\ -3.1865 & 4.1865 \end{bmatrix} \quad (4.2)$$

Where $P_{11}+P_{12}=1$, $P_{21}+P_{22}=1$

The probability of transitioning to expansion in the next period given that the current state is in expansion is 2.71745, probability of transitioning to a contraction in the next period given that the current state is in expansion is -1.71745. The probability of transitioning to expansion in the next period given that the current state is in contraction -3.1865. The probability of transitioning to a contraction in the next period given that the current state is in contraction 4.1865.

4.4 Expected Duration Inflation Rate

The expected time spent in each state is called the expected duration. If D_1 is the expected duration spent in state 1, is denoted as

$$E(D_1) = \frac{1}{1-p_{11}} = E_{11} \quad (4.3)$$

The closer P_{11} is to 1 the higher is the expected duration of state 1

Figure 4.1: Filtered Probability of Crude Oil Prices and Inflation Rate

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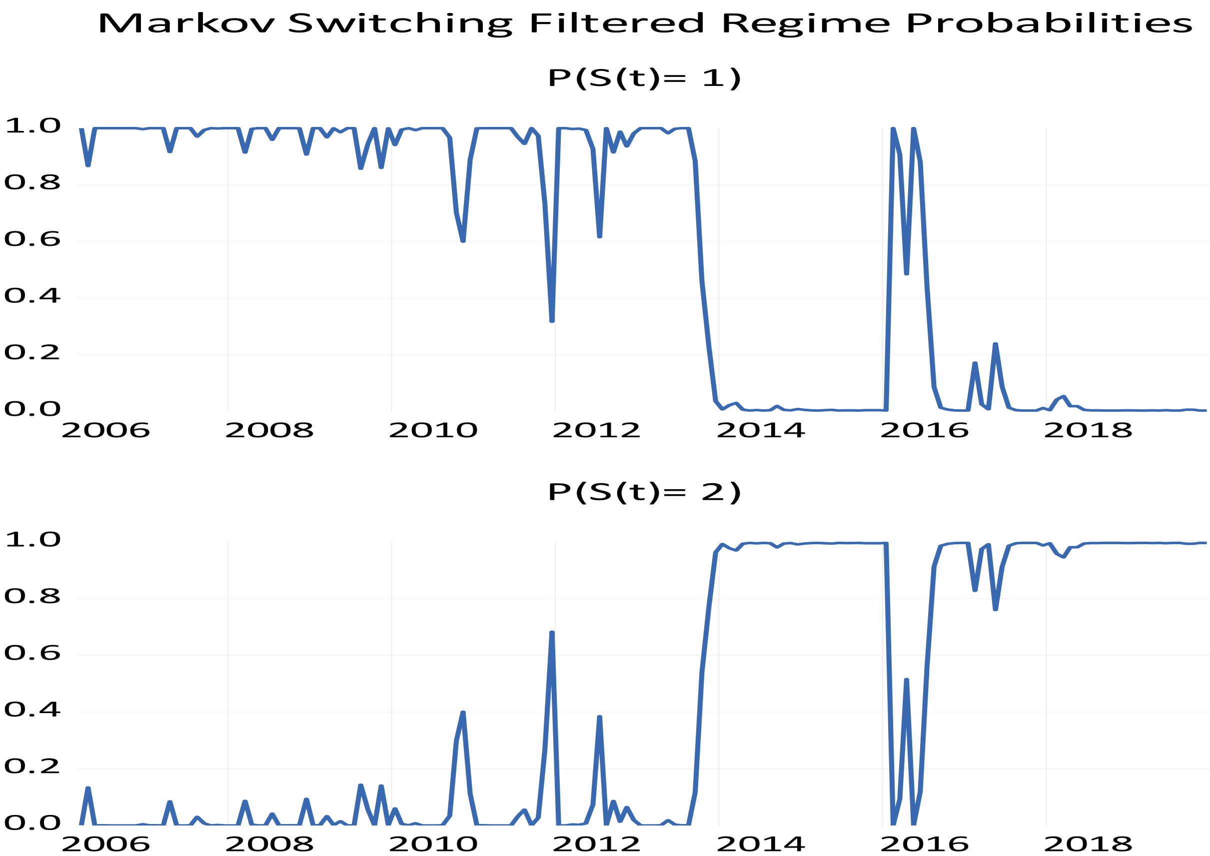

The filtered probabilities provide inference on s_t conditional on all available sample information, exogenous and endogenous switching process. In regime one $pr(s_t = 1)$ expansion, we find out that the filtering process is low from the being and low and high at the end, in regime two $pr(s_t = 2)$, contraction filtered is in opposite direction from the being and low to high and low at the end. However, during the financial crisis filtered is away in a high regime of variance which corresponds to a high level of market volatility.

4.5 Forecast Equation for 2020 January -2020 December at both Regime

At regime one (1) can be rewrite as;

January 2020

$$\begin{bmatrix} inf_t \\ cop_t \end{bmatrix} = \begin{pmatrix} 10.582965 \\ 81.858993 \end{pmatrix} + \begin{pmatrix} 0.32319 & -0.191735 \\ -0.000436 & 0.28562 \end{pmatrix} \begin{pmatrix} inf_{t-1} - 0.0506 \\ cop_{t-1} - 0.07803 \end{pmatrix} + \begin{pmatrix} 0.112679 & 0.285623 \\ 0.02076 & 0.082813 \end{pmatrix} \begin{pmatrix} inf_{t-2} - 0.05081 \\ cop_{t-2} - 0.07766 \end{pmatrix} \quad (4.4)$$

February 2020

$$\begin{bmatrix} inf_{t+1} \\ cop_{t+1} \end{bmatrix} = \begin{pmatrix} 10.582965 \\ 81.858993 \end{pmatrix} + \begin{pmatrix} 0.32319 & -0.191735 \\ -0.000436 & 0.28562 \end{pmatrix} \begin{pmatrix} inf_{t} - 0.0506 \\ cop_{t} - 0.07803 \end{pmatrix} + \begin{pmatrix} 0.112679 & 0.285623 \\ 0.02076 & 0.082813 \end{pmatrix} \begin{pmatrix} inf_{t-1} - 0.05081 \\ cop_{t-1} - 0.07766 \end{pmatrix} \quad (4.5)$$

December 2020

$$\begin{bmatrix} inf_{t+11} \\ cop_{t+11} \end{bmatrix} = \begin{pmatrix} 10.5829 \\ 81.8589 \end{pmatrix} + \begin{pmatrix} 0.32319 & -0.191735 \\ -0.000436 & 0.28562 \end{pmatrix} \begin{pmatrix} inf_{t+10} - 0.0506 \\ cop_{t+10} - 0.07803 \end{pmatrix} \begin{pmatrix} 0.112679 & 0.285623 \\ 0.02076 & 0.082813 \end{pmatrix} \begin{pmatrix} inf_{t+9} - 0.05081 \\ cop_{t+9} - 0.07766 \end{pmatrix} \quad (4.6)$$

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At regime two (2) can be rewrite as;

January 2020

$$\begin{bmatrix} inf_t \\ cop_t \end{bmatrix}=\begin{pmatrix} 10.42868 \\ 81.282818 \end{pmatrix}+\begin{pmatrix} 0.32319 & -0.191735 \\ -0.000436 & 0.28562 \end{pmatrix}\begin{pmatrix} inf_{t-1}-0.0506 \\ cop_{t-1}-0.07803 \end{pmatrix}+\begin{pmatrix} 0.112679 & 0.285623 \\ 0.02076 & 0.082813 \end{pmatrix}\begin{pmatrix} inf_{t-2}-0.05081 \\ cop_{t-2}-0.07766 \end{pmatrix} \quad (4.7)$$

February 2020

$$\begin{bmatrix} inf_{t+1} \\ cop_{t+1} \end{bmatrix}=\begin{pmatrix} 10.42868 \\ 81.282818 \end{pmatrix}+\begin{pmatrix} 0.32319 & -0.191735 \\ -0.000436 & 0.28562 \end{pmatrix}\begin{pmatrix} inf_{t}-0.0506 \\ cop_{t}-0.07803 \end{pmatrix}+\begin{pmatrix} 0.112679 & 0.285623 \\ 0.02076 & 0.082813 \end{pmatrix}\begin{pmatrix} inf_{t-1}-0.05081 \\ cop_{t-1}-0.07766 \end{pmatrix} \quad (4.8)$$

.December 2020

$$\begin{bmatrix} inf_{t+11} \\ cop_{t+11} \end{bmatrix}=\begin{pmatrix} 10.4286 \\ 81.2828 \end{pmatrix}+\begin{pmatrix} 0.32319 & -0.1917 \\ -0.0004 & 0.28562 \end{pmatrix}\begin{pmatrix} inf_{t+10}-0.0506 \\ cop_{t+10}-0.07803 \end{pmatrix}+\begin{pmatrix} 0.112679 & 0.28562 \\ 0.02076 & 0.082813 \end{pmatrix}\begin{pmatrix} inf_{t+9}-0.05081 \\ cop_{t+9}-0.07766 \end{pmatrix} \quad (4.9)$$

Table 4.3: Forecast Value of Inflation Rate and Crude Oil Prices in Nigeria

Month	Inflation rata	Crude oil price
January	11.99969	59.70189
February	12.12269	60.36689
March	12.88959	61.07625
April	13.93943	61.80165
May	15.1844	62.53495
June	16.53701	63.27185
July	17.9531	64.0104
August	19.56901	64.74897
September	20.81669	65.4893
October	22.22958	66.229
November	23.78186	66.969
December	25.27186	67.712

5.0 Conclusion

This paper examined the Markov-Switching Vector Autoregressive (MS-VAR) model developed by Hamilton (1989) and extended by Krolzig to capture Markov switching behaviour in the mean and the variance of Nigeria crude oil prices and inflation rate between January 2006 and December 2019. The switching behaviour of the mean and the variance of the study variables were examined, consequently, eight Markov-Switching Mean Vector Autoregressive Models were estimated. The AIC and SC test was used in model selection and both tests suggested that MSMH(2)-VAR(2) performed better than the rest models. The appropriate fitted model for the series, therefore, was MSMH (2)-VAR (2). All the processes were white noise. However, the filtered probability provided inference on s_t conditional on all available sample information. The best model was also used to forecast the value of the series from January 2020 to December 2020.

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