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PROPAGATION PATH LOSS IN RAIN MEDIA USING PARABOLIC EQUATION METHOD

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ABSTRACT: A model to predict millimeter wave (mmW) propagation path loss through rain medium environment using parabolic equations method (PEM) is reported. For the solution of PEM, the Split-Step Fourier Transform (SSFT) algorithm is used to investigate path loss of mmW propagating in rain medium with irregular terrain with low-grazing angle or near horizontal propagation. The model consists of series of phase screens through which millimeter wave signal propagates. Finally, the model is applied to simulate the propagation characteristics of millimeter wave in rain over irregular terrain.

Keywords: Parabolic equations, Split-step Fourier transform, millimeter wave propagation, path loss.

1. INTRODUCTION

Modern cities' infrastructural development and financial success have been greatly impacted by wireless communication. Global satellite linkages and regional cellular networks are made possible by it. Wireless communication are created to satisfy public and industrial requirements, and to also support high data transmission rates and network node connectivity due mainly to advert of internet of things. Larger bandwidths and low latency millimetre wave (mmW) wireless technologies are being utilized in the 5G mobile service era to meet these demands, and to improving the overall quality of the service [1]. This enhances network efficiency, throughput, and spectral efficiency for reliable service delivery.

However, the quality of propagating mmW signals is affected mainly by meteorological variables such as rain, fog and snow [2]. The propagating signal energy attenuates and a reduced signal coverage area results due to path loss. The path loss is the loss of signal energy during propagation from transmitter to receiver. To precisely develop the desired propagation model of mmW in rain media necessary to obtain effective predictions of path loss of a system of radio, we make some important assumptions. Here, we adopt Debye model for the estimation of the effective dielectric constant of water and consider a linear, isotropic non-ionized medium, and the electrical properties of this medium were modelled as a lossy dielectric [3].

Millimeter wave propagation in rainfall environment has been a research subject for a long time, due to weakling strength signals with increasing distance. Recent interests in this subject have been aroused in the wake of the development of wireless communication systems through consideration of high directivity necessary to strengthen the trans-receiver link. Various numerical techniques are in use to model the propagation of electromagnetic waves: asymptotic and rigorous methods. In rigorous methods such as the finite-difference time-domain (FDTD) method, the finite element method (FEM), and the method of moments (MoM) [1], are exact and not approximate equations derived from Maxwell's equations and numerically solved. Asymptotic methods, such as ray tracing, physical optics (PO), parabolic equation (PE), and Gaussian/wavelet-based methods are solutions of approximated equations, and better suited for long distances propagation modeling [4]. The Split- Step Fourier (SSF) and the implicit Finite-Difference (FD) are commonly used for resolving parabolic wave equations problems [1]. SSF offers advantages such as modelling refractivity in the spatial domain by using phase-screens method, development of various algorithms for modelling of irregular relief, and a spectral transform boundary condition for consideration of the ground composition [5]. The model presented in this article is focused on using the scheme of Split-step Fourier algorithm to solve parabolic equation and to the analysis of millimeter wave diffraction and refraction by raindrops [6]. Our purpose is to calculate the loss along the propagation path in a rain environment, and propagation is considered 15° in the paraxial direction. Here, we used the complex refractive index of water and assume the medium to be linear, isotropic and non-ionized with a lossy dielectric. The propagation model is described in the section 2, in the section 3, the method of split-step Fourier transform is described and the results are presented in the section 4.

2. MODEL OF PROPAGATION

Consider a simple model of millimeter wave propagating in the *xz*-plane with component ψ as in Figure 1, and wave equation is

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k^2n^2\right)\psi=0 \qquad (1)$$

where $x =$ the longitudinal (the direction of propagation) coordinate, $z =$ the transverse (the height above the ground) coordinate, n the refractive index, and $k_0 = {}^{2\pi}/_{\lambda}$ the free space wave number [7]. For a homogeneous propagating medium, ψ satisfies scalar wave equation in two dimensions:

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}+k^2n^2\psi=0 \qquad (2)$$

Introducing the reduced function, $u(x,z)$ associated with the paraxial direction of propagation x

$$u(x,z)=\psi(x,z)e^{-ikx} \qquad (3)$$

where u denotes the wave amplitude (either of the electric or magnetic field components depending on the type of the problem).

Writing equation (2) in function of equation (3), the PWE becomes

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}+2ik_0\frac{\partial u}{\partial x}+k_0^2(n^2-1)u=0 \qquad (4)$$

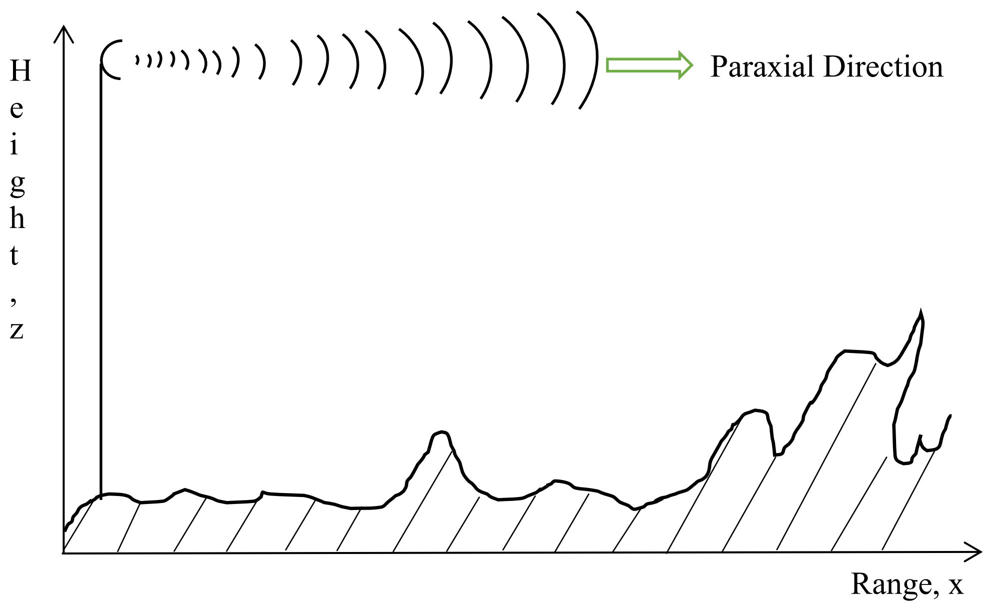

Figure 1: Sketch of a Propagating Parabolic Waveform

Assuming that the refractive index $n(x,z)$ possess smooth variations, $\frac{\partial^2}{\partial z^2}$ becomes negligible (paraxial approximation) and equation (4) becomes

$$\frac{\partial^2 u}{\partial x^2}+2ik_0\frac{\partial u}{\partial x}+k_0^2(n^2-1)u=0 \qquad (5)$$

Using method of separation of variables, equation (5) can be written as

$$\frac{\partial u}{\partial x}=\frac{ik}{2}\left\{\frac{1}{k^2}\frac{\partial^2}{\partial x^2}+(n^2(x,z)-1)\right\}u=iMu \qquad (6)$$

Where $M = (1-n^2)\frac{k_0}{2} - \frac{p^2}{2k_0}$.

The analytic solution of the parabolic wave equation becomes

$$u(x_0 + \Delta x, z) = u(z_0)e^{iM\Delta x} \quad (7)$$

It is worthy of note that the equation (5) can be factored into two terms using the factorization [7] to obtain

$$\left\{\frac{\partial}{\partial x} + ik(1-Q)\right\}\left\{\frac{\partial}{\partial x} + ik(1+Q)\right\}u = 0 \quad (8)$$

Where Q is used to defined pseudo-differential operator as

$$Q = \sqrt{\frac{1}{k^2}\frac{\partial^2}{\partial x^2} + n^2(x,z)} \quad (9)$$

The first bracket in equation (8) represents the progressive propagation of parabolic wave equation, solved separately by adopting paraxial approximation. For example, solving for energy propagating in a paraxial cone centered on the positive x-direction as in Figure 1.

3. METHOD OF SPLIT-STEP FOURIER TRANSFORM SOLUTION (SSFT)

The split-step Fourier transform is a solution of parabolic wave equation method and very efficient in area of decoupling the refractive effect from the diffractive part of the propagator. Hardin and Tappert [6] solved the problem of modelling ionospheric radar propagation by developing the Split-step Fourier parabolic equation (SSFPE) algorithm. Due mainly to advances in computational technology and evolution, the technique gained prominence in relation to other numerical techniques, and has been of great usefulness in radar propagation to study anomalous microwave propagation in the troposphere [8], applied by Dockery and Konstanzer to analyse phased radar performance, and recently, several authors have developed electromagnetic PE models [1].

In this article, the split-step Fourier Transform of parabolic equation method is used, which allows the modeling of propagating millimeter wave in range-dependent environment as series of phase screens. Considering a two-dimensional scalar wave equation for horizontally and vertically polarized wave, the split-step Fourier method transforms the rough surface problem with propagation through a sequence of phase screens.

Assuming that the refractive index $\boldsymbol{n}$ is range independent but varies only height $\boldsymbol{y}$, and let

$$A = \frac{1}{k^2}\frac{\partial^2}{\partial x^2} \quad (10)$$

$$B = n^2(x,z) - 1 \quad (11)$$

Equation (6) becomes

$$\frac{\partial u}{\partial x} = \frac{ik}{2}\{A+B\}u \quad (12)$$

The analytic solution of the SPE therefore becomes

$$u(x+\Delta x, z) = u(x,z)e^{\delta(A+B)} \quad (13)$$

where $\delta = \frac{ik\Delta x}{2}$ (14)

Equation (13) is the split-step solution, visualized as a field propagating through series of phase screens of two distinct regions in a homogeneous medium modulated by the refractive index variations. We have the propagator for the narrow-angle that takes the solution from $x + \Delta x$, given as:

$$u(x+\Delta x, z) = exp\left(\frac{ik\Delta x}{2}\left[n^2\left(z+\frac{\Delta x}{2}, z\right) - 1\right]\right)\mathfrak{F}^{-1}\left\{e^{\left(-\frac{i\Delta z p_x^2}{2k}\right)}\right\}\mathfrak{F}\{u(x,z)\} \quad (15)$$

If we consider a PWE source with antenna Gaussian, beam pattern defined as

$$f(p) = \exp\left(\frac{-p_x^2 2ln2}{4\left(k_0 \sin\left(\theta_{bw}/2\right)^2\right)}\right) \quad (16)$$

For far-field antenna pattern specified with height z_0, antenna beamwidth θ_{bw} and tilt angle θ_{tilt}, the initial field profile $U(0,p)$ in transverse wave number (p) domain can be obtained using inverse Fast Fourier transform, which obeys Dirichlet and Neumann boundary conditions. $U(0,p)$ is the forward Fourier transform of $u(x_0, z)$, while $u(z0, x)$ is the initial field profile and incident propagating field of the PE which we can describe as the field at range $x = 0$. Fourier transform solves equation (6) by direct converting the field profile from its spatial form (z)−domain to spectral form (p)−domain. For a rain medium, the refractive index can be assumed constant with respect to z for each small range step size Δx.

The numerical split-step parabolic equation solution for $j = 1, 2, \ldots M$ is therefore given as

$$u(x_0 + j\Delta x, z) = exp\left[\frac{ik_0}{2}(n^2 - 1)\Delta x\right] \mathfrak{F}^{-1}\left\{e^{\left(-\frac{i\Delta z p_x^2}{2k}\right)}\right\} \mathfrak{F}\{u(x_0 + (j-1)x, z)\} \quad (17)$$

where $\mathfrak{F}$ and $\mathfrak{F}^{-1}$ are the forward and inverse Fourier transforms; $p = k_0 sin\,\alpha$ is the transform variable, and α is the propagation angle relative to the horizontal. For a given initial field profile $u(0, z)$, $u(x + \Delta x, z)$ is calculated along the x -axis with the steps of Δx.

4. RESULTS AND DISCUSSION

Table 1 shows rainfall data collected for the city of Yenagoa in Southern Nigeria. Since the rain rate less than 1 mm/h and diameter less than 2 mm, the shape of the raindrop is spherical [8]. We apply the PE method to model the propagation of millimeter wave in rain over irregular terrain with average temperature of about 25 ^0C and average rain rate 0.4266 mm/h as shown in Table 1.

Table 1. Yenagoa Climate Weather Averages for 2024 [9]

Month	Temperature (◦C)	Rain Days *(Day)*	Monthly Rain Amount (mm)	Hourly Rain Rate (mm/h)
January	28	7	16.90	0.0227
February	28	8	12.11	0.0174
March	26	3	41.37	0.0556
April	26	25	359.85	0.4998
May	26	24	224.85	0.3022
June	24	28	688.50	0.9563
July	24	31	691.85	0.9299
August	24	24	638.80	0.8586
September	23	17	525.11	0.7293
October	24	10	352.44	0.4737
November	25	8	171.02	0.2375
December	27	1	26.61	0.0358

Adopting Debye equation, the effective permittivity of water for a given operating frequency is given as [10].

$$n = \sqrt{\epsilon} = \left[\epsilon' - i\frac{\sigma}{2\pi f \epsilon_0}\right]^{1/2} \quad (18)$$

where $\boldsymbol{n}$ is the refractive index, $\boldsymbol{\epsilon}$ is the complex permittivity, σ is the conductivity (S/m), $\boldsymbol{f}$ is the frequency (Hz) and $\boldsymbol{\epsilon_0}$ is the permittivity in free space (F/m).
Equation (19) calculate the path loss, L.

$$L = 10log\frac{(4\pi a)^2}{f^2 D_t D_r} e^{(N(d)\sigma_{total})a} \quad (19)$$

where $N(d)$ is the density per unit volume of the rain drops, σ_{total} is the total cross section of the scattering and absorption, D_t and D_r the maximum directive gains in dB of the transmitting and receiving antennas and respectively, f the frequency in GHz of the radio signals, and $\boldsymbol{a}$ is the distance apart of the transmit and the receive antennas.

Now, consider a Gaussian source field that is horizontally and vertically polarized at $z_s = 100\ m$, with a beamwidth of 1°, elevation or tilt angle 1°, and frequency $300\ GHz$. The surfaces of the terrains and ground are assumed to be wet field profile over range as the field travels through flat earth through rain medium with average rain rate $0.4266 mm/h$, average temperature 25.4^{0}C and effective refractive index $n = 2.58125 + 1.1304i$ with maximum heights $5\ km$ and $10\ km$, earth radius $r_e = 6371\ km$. The simulation was implemented using numerical split-step PE solution equation (17) for $j = 1, 2, \dots M$.

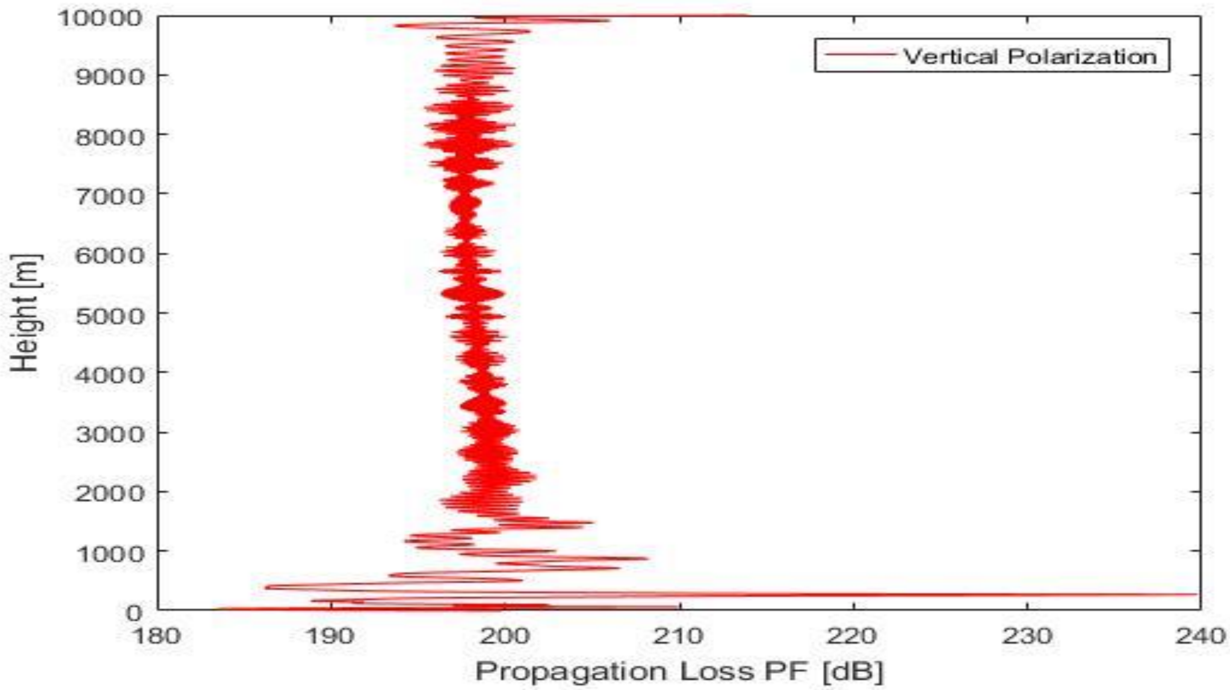

Figure 2: Propagation loss versus height over a flat earth with rain rate $0.4266\ mm/h$

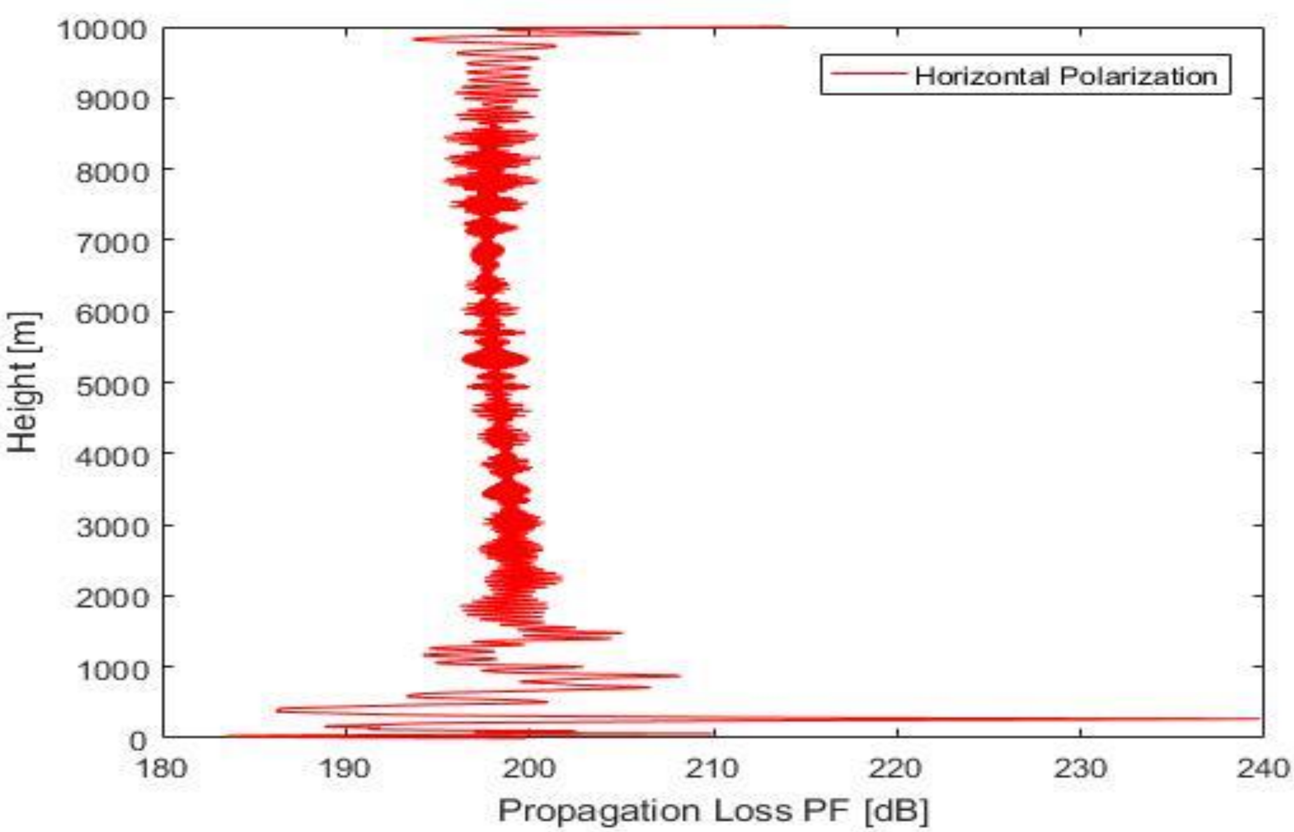

Figure 3: Propagation loss versus height over a flat earth with rain rate 0.4266 mm/h

Figures 2 and 3 illustrate the propagation loss as a function of height in rain medium with maximum height $z = 10\ km$ for both horizontal and vertical polarizations. The field profiles vary similarly in both cases for maximum range $x = 50\ km$. The flat earth surfaces show less diffuse reflections than irregular terrains assumed wet as shown in both cases of horizontal and vertical polarizations. Figures 4 and 5 illustrate the propagation loss as a function of range in rain medium with maximum height $z = 10\ km$ for both horizontal and vertical polarizations. The field profiles vary similarly in both cases for maximum height $z = 10\ km$. The flat earth surfaces show less diffuse reflections than irregular terrains assumed wet as shown in both cases of horizontal and vertical polarizations. The propagation loss is highest at range $x = 800\ km$ and lowest at $x = 0\ km$. This explains that low altitudes, multipath propagation effects are not significant.

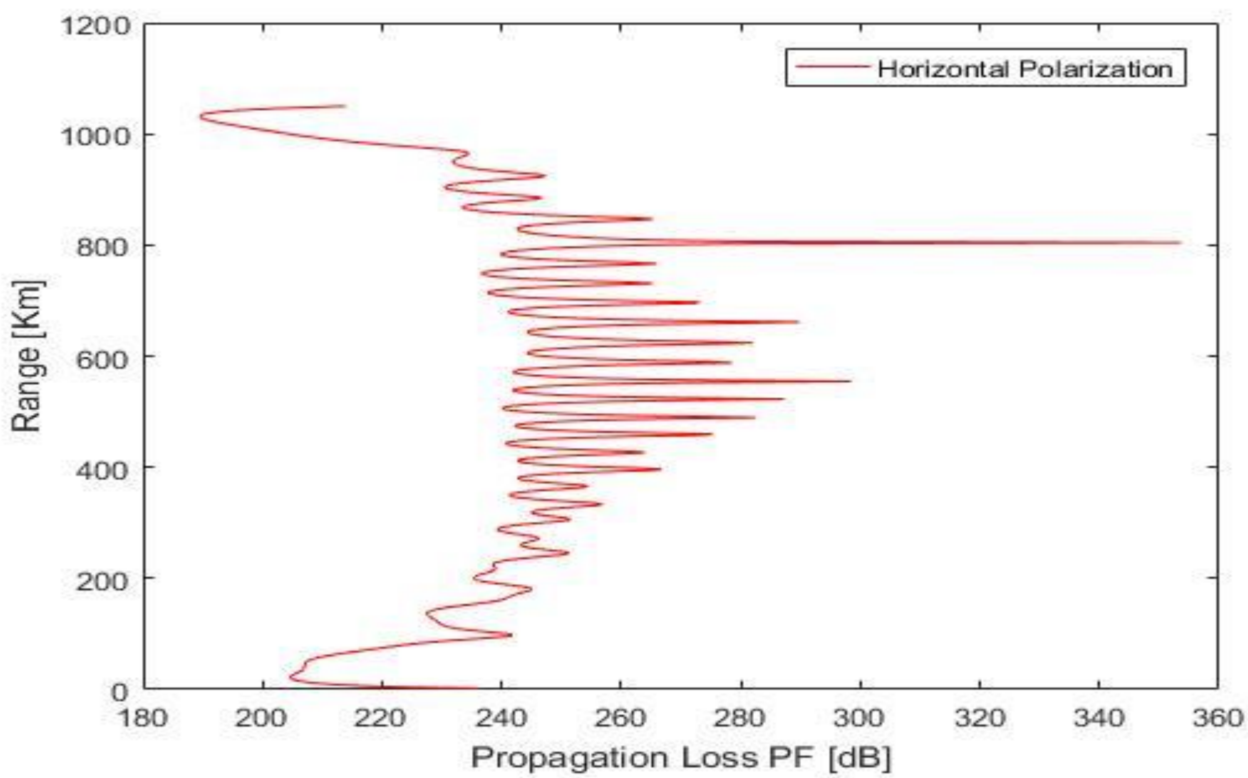

Figure 4. Propagation loss versus Range over a flat earth with $R = 0.4266\ mm/h$

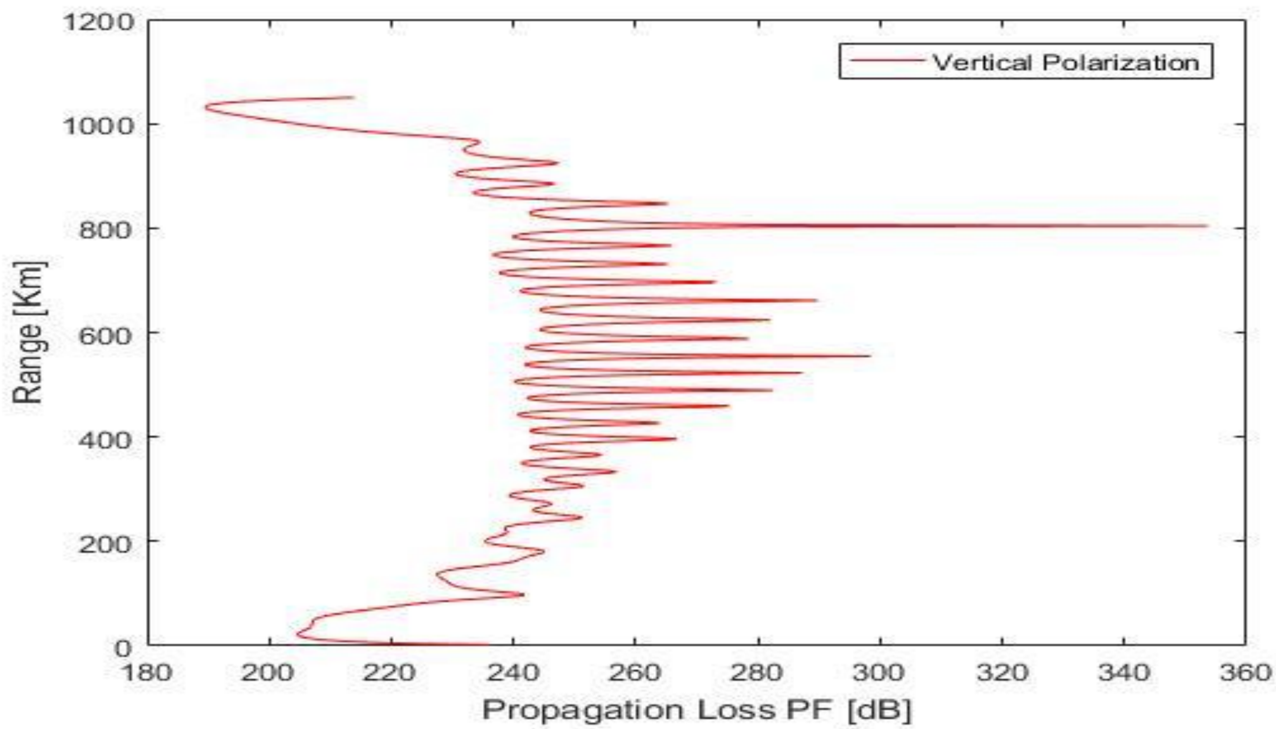

Figure 5. Propagation loss versus Range over a flat earth with $R = 0.1584\, mm/h$

Figures 2 - 5 illustrate the propagation loss, L in rain medium for vertical and horizontal polarizations. We observed that the rain causes strong propagation loss at the frequency of 300 GHz. It verifies that the PE method can handle multipath propagation.

5. CONCLUSION

The study of millimeter-wave propagation in rain is of great significance for practical engineering applications. The parabolic equation model was applied to simulate mmW propagation loss caused by raindrops in rain medium with irregular terrain conditions. The results demonstrate that PE model can predict multipath propagation effects; and the Split step Fourier method is suitable for simulating long-range millimeter-wave propagation with complex geographical and meteorological conditions.

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