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FRACTIONAL MODEL ON THE TRANSMISSION DYNAMICS OF COMPUTER VIRUS

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Abstract: This paper studies the fractional order model for computer virus in SIRA model, with the antidotal class incorporated. Firstly, the basic reproduction number, R_0, which determines the threshold of the spread of the virus is calculated. The stability of the equilibria was also investigated and studied. To illustrate our theoretical analysis, some numerical simulations were carried out, using Adams-type predictor-corrector method, to show the behavior of the solutions of the proposed fractional order system. The results provide a theoretical basis to control the spread of computer virus. The simulations, also, showed that the fractional model gives a better result than the integer models.

Keywords: Computer Virus; Antidotal class; Fractional-order differential equations; Equilibrium points; Stability; Predictor-corrector method.

1. Introduction

Computer viruses and network worms, are defined as malicious codes that can replicate themselves and spread among computers [1]. The spread of computer viruses still cause enormous financial losses and computer security problems to large organizations [2]. The most devastating computer virus to date referred to as " My Doom", has caused over $38 billion in damages to individuals and organizations [3]. In the past two decades, computer viruses were inherently limited by human mediation required for their propagation [5]. However, human intervention in recent times have played a significant role in preventing the outbreak of computer viruses [6]. Myriad of different computer viruses have been developed to damage computer systems, erasing data or stealing information [7]. Computer virus is not only destructive, but also highly contagious. Once the virus is copied or generates variety, its spread is difficult to control. Infectivity is the basic characteristic of computer virus. In biology, a virus can diffuse from one organism to another. Cohen, Kephart and White [8, 9] pointed out the similarities between the spread mechanisms of computer and biological viruses. Under appropriate conditions, biological viruses can multiply quickly, exhibiting symptoms in infected organism which may lead death. Similarly, computer virus can spread among uninfected computers in a couple of different of ways such as

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mobile hard disk, email attachments, during file downloads from internet or a visit to virus contaminated website. In some cases, the infected computers develop different dysfunctionalities and may not work. Unlike biological viruses, computer virus is a software designed to replicate itself and spread amongst machines. Once it enters the computer and gets executed, it will search for, and target programs or storage media with suitable environmental conditions for the purpose of self-reproduction. As long as the computer remains infected, the virus will continue to spread rapidly infecting a large number of executable files. These infected files will become a new source of infection to other computer connected over the network or during data exchanges. It will remain contagious if not treated. Computer network has no permanent immunity to computer virus. Therefore, there are always computer virus [10].

El-Sayed et al [11] proposed a fractional order SIR model for computer virus transmission under human intervention. Ebenezar et al [15] present a fractional SEIR model to explain the dynamics of computer propagation. Q. Peng [10] proposed an integer SIRA model to explain the dynamics of computer virus.

The approach presented in this paper differ from those presented in [10] and references therein. We present a fractional order SIRA (Susceptible-Infected-Removed-Antidotal) model to discuss the dynamics of computer virus with new computers incorporated into the network, and old computers removed. The major reason for considering fractional differential equations is beacuse they are naturally related to systems with memory which exists in both biological and dynamical systems [11].

This paper is organized as follows. A brief review of the fractional calculus is presented in section 2 with definitions. Section 3 discusses fractional order models while section 4 presents model analysis involving equilibrium points and stability. Section 5 is devoted to numerical simulations and discussion of results. Section 6 gives the concluding remarks.

2. Fractional Order Calculus

Fractional order models have been the focus of many studies due to their frequent appearance in various applications in several scientific fields. We first give the definition of fractional-order integration and fractional order differentiation from [12]. For the concept of fractional derivative, we will consider Caputo's definition. It has an advantage of dealing properly with initial value problem [12].

2.1 Definition of terms

Definition 1. The Caputo Fractional derivative of order α of a function $f: <^{+} \rightarrow <$ is given by

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha - n)} \int_{\alpha}^{t} \frac{f^{(n)}(\tau) d\tau}{(t-\tau)^{\alpha+1-n}} \qquad (n-1 < \alpha \leq n) \qquad (2.1)$$

Definition 2. The formula for the Laplace transform of the Caputo derivative is given by

$$\int_0^{\infty} e^{-pt} \{D_t^{\alpha} f(t)\} dt = p^{\alpha} F(p) - \sum_{k=0}^{n-1} p^{\alpha-k-1} f^{(k)}(0), \qquad (n-1 < \alpha \leq n) \qquad (2.2)$$

Definition 3. The Fractional integral of order α of a function $f: <^{+} \rightarrow <$ is given by

$$J^{\alpha}(f(x)) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t) dt, \qquad \alpha > 0, x > 0 \qquad (2.3)$$

Definition 4. The fractional integral of the Caputo Fractional derivative of order α of a function $f: <^{+} \rightarrow <$ is given by

$$J^{\alpha}\{D^{\alpha} f(t)\} = f(t) - \sum_{k=0}^{n-1} f^{(k)}(0) \frac{t^k}{k!}, \qquad t > 0 \qquad (2.4)$$

Definition 5. A two-parameter funvtion of the Mittag-Leffler type is defined by the series expansion

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \qquad (\alpha, \beta > 0) \qquad (2.5)$$

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3. Model Formulation

We propose a model based on the compartmental SAIR model [10], including the antidotal population compartment (A) representing nodes of the network equipped with fully effective antivirus programs and was then converted to fractional. A susceptible compartment (S), an infective compartment (I), and a temporarily immune compartment (R), were considered. The total population N is divided into four groups, $S(t)$ denote noninfected computers subjected to possible infection. $I(t)$ denote infected computers. $R(t)$ denote removed computers due to infection or not. $A(t)$ denote noninfected computers equipped with antivirus. The model is formulated as the following system of differential equations. The fractional SAIR model is described by

$$\begin{aligned} {}^{\square}D^{\alpha}S(t) &= \Lambda - \beta SI - \delta SA - \mu S + \omega R \\ {}^{\square\square}{}_{\square}D^{\alpha}I(t) &= \beta SI - \gamma I - \mu I - \theta IA \\ (3.1)\ {}^{\square\square\square}DD^{\alpha\alpha}RA((tt) =) &= \delta SA\gamma I - +\omega R\theta IA - \mu R - \mu A \end{aligned}$$

Λ represents the recruitment rate of Computers into the network, β represents the rate at which Susceptible (S) becomes infected (I), γ represents rate at which the infected (I) becomes recovered (R), δ represents the rate at which Susceptible (S) becomes Antidotal (A), θ represents the rate at which infected/infectious (I) becomes Antidotal (A), ω represents the rate at which recovered (R) becomes Susceptible (S). The model flow diagram is as shown below:

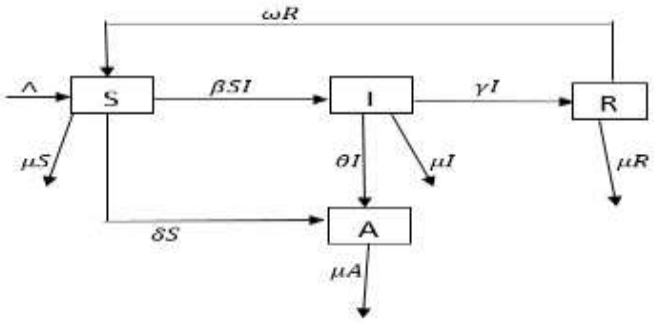

Figure 1: Model Flow Diagram

3.1 Invariant Region

An invariant region for a model is a region where the property of the mathematical model remains unchanged, after operations or transformations of a certain type are applied to the model. The lemma below shows the region where the solution of the model (3.1) is valid.

Lemma 3.1. The closed set $\Omega = \{(S, I, R, A) \in R_+^4 : S + I + R + A \leq \frac{\Lambda}{\mu}\}$ is positively invariant with respect to model (3.1)

Proof: The fractional derivative of the total human population, obtained by adding all the human equations of model (3.1), is given by

$$D^{\alpha}N(t) = \Lambda - \mu N(t) \qquad (3.2)$$

Taking the Laplace transform of (3.2) gives:

$$\begin{aligned} S^{\alpha}N(s) - S^{\alpha-1}N(0) &= \frac{\Lambda}{S} - \mu N(s) \\ \Rightarrow N(s) &= \frac{\Lambda}{S(S^{\alpha}+\mu)} + \frac{S^{\alpha-1}}{s^{\alpha}+\mu}N(0) \end{aligned} \qquad (3.3)$$

Taking the inverse Laplace transform of (3.3), we have:

$$N(t) = N(0)E_{\alpha,1}(-\mu t^{\alpha}) + \Lambda t^{\alpha}E_{\alpha,\alpha+1}(-\mu t^{\alpha}) \qquad (3.4)$$

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where $E_{\alpha,\beta}$ is the Mittag-Leffler function. But the fact that the Mittag-Leffler functions has an asymptotic behavior [11, 12], it follows that:

$$E_{\alpha,1}N(t) = \sum_{k=0}^{\infty} \frac{N^k(t)}{\Gamma(\alpha k+1)}, \alpha > 0 \quad (3.5)$$

$$E_{\alpha,\alpha+1}N(t) = \sum_{k=0}^{\infty} \frac{N^k(t)}{\Gamma(\alpha k+\alpha+1)}, \alpha > 0 \quad (3.6)$$

Expanding (3.10), we have

$$E_{\alpha,1}N(t) = \frac{1}{\Gamma 1} + \frac{N(t)}{\Gamma(\alpha+1)} + \frac{N^2(t)}{\Gamma(2\alpha+1)} + \ldots$$

Expanding (3.11), we have

$$E_{\alpha,\alpha+1}N(t) = \frac{1}{\Gamma(\alpha+1)} + \frac{N(t)}{\Gamma(2\alpha+1)} + \frac{N^2(t)}{\Gamma(3\alpha+1)} + \ldots$$

Since Mittag-Leffler function has an asymptotic property, we have

$N(t) = 1 + O(N)$

Taking limit as k⟶ ∞, we have

$N(t) \approx 1$

Then, it is clear that Ω is a positive invariant set.Therefore, all solutions of the model with initial conditions in Ω remain in Ω for all t > 0. Then , $\Omega = N(t) > 0$ implies that it is feasible with respect to model (3.6).

4. Existence and Uiqueness of Solutions

Let $x_1(t) = S(t), x_2(t) = I(t), x_3(t) = R(t), x_4(t) = A(t)$ then system (3.1) can be written as:

$f_1(x_1(t),x_2(t),x_3(t),x_4(t)) = \Lambda - \beta x_1(t)x_2(t) - \delta x_1(t)x_4(t) - \mu x_1(t) + \omega x_3(t)$

$f_2(x_1(t),x_2(t),x_3(t),x_4(t)) = \beta x_1(t)x_2(t) - \gamma x_2(t) - \mu x_2(t) - \theta x_2(t)x_4(t)$

(4.1)

$f_3(x_1(t),x_2(t),x_3(t),x_4(t)) = \delta x\gamma x_1 2((tt))x - {}_4(\omega x t)_{+3}(t\theta x) - {}_3(_t\mu x)_x 3_4((tt)) - \mu x_4(t)$

Then on D, we have

$$\begin{cases} |\frac{\partial}{\partial x_i} f_1(x_i)| \le k_i & i = 1,2,3,4 \\ |\frac{\partial}{\partial x_i} f_2(x_i)| \le k_i & i = 1,2,3,4 \\ |\frac{\partial}{\partial x_i} f_3(x_i)| \le k_i & i = 1,2,3,4 \\ |\frac{\partial}{\partial x_i} f_4(x_i)| \le k_i & i = 1,2,3,4 \end{cases} \quad (4.2)$$

Where k_is are positive constants.

This implies that the functions f_1, f_2, f_3, f_4, satisfy the Lipchitz condition with respect to the arguments x_i, i = 1,2,3,4, then each of the functions f_1, f_2, f_3, f_4, is absolutely continuous with respect to the arguments x_i, i = 1,2,3,4,.

Consider the following initial value problem which represents the fractional-order SIRA model:

$$\begin{cases} D^\alpha x_1(t) = f_1(x_1,x_2,x_3,x_4), & t > 0, \quad x_1(0) = x_0^{(1)} \\ D^\alpha x_2(t) = f_2(x_1,x_2,x_3,x_4), & t > 0, \quad x_2(0) = x_0^{(2)} \\ D^\alpha x_3(t) = f_3(x_1,x_2,x_3,x_4), & t > 0, \quad x_3(0) = x_0^{(3)} \\ D^\alpha x_4(t) = f_4(x_1,x_2,x_3,x_4), & t > 0, \quad x_4(0) = x_0^{(4)} \end{cases} \quad (4.3)$$

Definition 6. By a solution of the fractional-order SIRA model (4.3), we mean a column vector $(x_1,x_2,x_3,x_4)^\tau$, x_1,x_2,x_3 and $x_4 \in C[0,T]$, $T < \infty$, where $C[0,T]$ is the class continuous functions defined on the interval $[0,T]$ and s denotes the transpose of the matrix.

Theorem 4.1. The fractional-order model (4.3) has a unique uniformly Lyapunov stable solution.

Proof: Write the model in the matrix form

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$$D^{\alpha}X(t) = F(x(t)), \quad t > 0 \quad and \quad x(0) = x_0. \tag{4.4}$$

where $x(t) = (x_1,x_2,x_3,x_4)^{\tau}$
and

$F(x(t)) = ((f_1(x_1,x_2,x_3,x_4))(f_2(x_1,x_2,x_3,x_4))(f_3(x_1,x_2,x_3,x_4))(f_4(x_1,x_2,x_3,x_4)))^{\tau}$

5. Model Analysis

In this section, the threshold parameter was calculated and was thereafter used to study the various equilibrium points of the system. This is relevant to the work, because it tells us whether or not the system would be stable after the small pertubation has been introduced

5.1 The Basic Reproduction Number, R_0

It is the average number of secondary infection/cases of the virus spread made by introducting a typical infections computer (during infection period) into a completely susceptible population.
Nnaemeka et al [13] computations of R_0 typically involve the product of infection.
R_0 = (Rate of secondary infection)∗(duration of infection) Reproduction number $R_0 = \rho(FV^{-1})$ where ρ denotes the spectral radius of a matrix V which represents the dominant nonnegative eigenvalue of the next generation matrix of the square matrices F and V of order
(m by m) where m is the number of infected class, defined by

$$F = \frac{\partial f_i}{\partial x_j}(x_0) \quad and \quad \frac{\partial v_i}{\partial x_j} = (x_0)$$

such that F is non-negative, V is a nonsingular m-matrix and x_0 is the disease free equilibrium point (DFE).
Now considering the infected classes of system (3.1), we have, $D^{\alpha}I(t) = \beta SI - \gamma I - \mu I - \theta IA = \beta SI - (\gamma - \mu - \theta)I = [\frac{\beta S}{(\gamma+\mu+\theta A)} - 1]I(\gamma + \mu + \theta A)|_{DFE,(\frac{\Lambda}{\mu},0,0,0)}$. Therefore, basic reproduction number R_0, of the model ($\alpha = 1$) is computed as

$$R_0 = \frac{\beta\lambda}{\mu(\gamma + \mu)}$$

For $R_0 < 1$, the disease-free equilibrium is globally asymptotically stable and if $R_0 > 1$, the endemic equilibrium is globally asymptotically stable.

5.2 Equilibrium Points and Stability

5.2.1 Disease free equlibrium point of the model

At DFE, there is no virus and as such no computer is infected, no computer recovered and there is no reason to make any computer antidotal, this implies that $I = R = A = 0$. The disease free equilibrium (DFE) of the virus is obtained by setting

$$D^{\alpha}S(t) = D^{\alpha}R(t) = D^{\alpha}A(t) = 0.$$

The system (3.1) becomes:

$$\square 0 = \Lambda - \beta SI - \delta SA - \mu S + \omega R$$
$$\square^{\square\square}{}_{\square} 0 = \beta SI - \gamma I - \mu I - \theta IA$$
$$(5.1)\ \square^{\square\square\square} 0 = 0 = \gamma I \delta SA - + \omega R \theta IA - \mu R - \mu A$$

On solving the right hand side of equation gives the disease free state of the model. Thus, the DFE is given by $E^0(S^0, I^0, R^0, A^0) = (\frac{\Lambda}{\mu}, 0, 0, 0)$.

5.2.2 Local Stability of the Disease-free Equlibrium Point

The Jacobian matrix $J(E^0)$ for the fractional order model (5.1) computed at E^0 is given as:

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$$J(E^0) = \begin{pmatrix} \frac{\partial S}{\partial S} & \frac{\partial S}{\partial I} & \frac{\partial S}{\partial R} & \frac{\partial S}{\partial A} \\ \frac{\partial I}{\partial S} & \frac{\partial I}{\partial I} & \frac{\partial I}{\partial R} & \frac{\partial I}{\partial A} \\ \frac{\partial R}{\partial S} & \frac{\partial R}{\partial I} & \frac{\partial R}{\partial R} & \frac{\partial R}{\partial A} \\ \frac{\partial A}{\partial S} & \frac{\partial A}{\partial I} & \frac{\partial A}{\partial R} & \frac{\partial A}{\partial A} \end{pmatrix}$$

This gives

$$J(E^0) = \begin{pmatrix} -(\beta I + \delta A + \mu) & -\beta S & \omega & -\delta S \\ \beta I & \beta S - (\gamma + \mu + \theta)A & 0 & -\theta I \\ 0 & \gamma & -(\omega + \mu) & 0 \\ \delta A & \theta A & 0 & \theta I - \mu \end{pmatrix}$$

Now evaluating the Jacobian at DFE, we obtain

$$J(E^0) = \begin{pmatrix} -\mu & -\frac{\beta\Lambda}{\mu} & \omega & -\frac{\delta\Lambda}{\mu} \\ 0 & \frac{\beta\Lambda}{\mu} - (\gamma + \mu) & 0 & 0 \\ 0 & 0 & -(\omega + \mu) & 0 \\ 0 & 0 & 0 & \frac{\delta\Lambda}{\mu} - \mu \end{pmatrix} \quad (5.2)$$

Theorem 5.1. The equilibrium point E^0 of system (5.1) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$

Proof: The equilibrium point E^0 is locally asymptotically stable if all the eigenvalues λ_i, $i = 1,2,3,4$ of $J(E^0)$ satisfy the condition [13]: $|arg(\lambda_i)| > \frac{\alpha\pi}{2}$.

From the Jacobian matrix $J(E^0)$, it is clear that $\lambda_1 = -\mu$, $\lambda_2 = -(\omega + \mu)$, $\lambda_3 = \frac{\delta\Lambda}{\mu} - \mu$ and $\lambda_4 = \frac{\beta\Lambda}{\mu} - (\gamma + \mu)$. If $\frac{\delta\Lambda}{\mu} < \mu$ and $R_0 < 1$, then all the eigenvalues λ_i satisfy the condition $arg(\lambda_i) > \frac{\alpha\pi}{2}$. Hence, the disease-free equilibrium is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

5.2.3 Endemic equlibrium points and their stabilities

Here, we considered two endemic equlibrium point E_1^* and E_2^*

1. When $A = 0$, we have that

According to the second system of equation (3.1), we have

$$S^* = \frac{\gamma + \mu}{\beta} \quad (5.3)$$

According to the third system of equation (3.1), we have

$$R^* = \frac{\gamma}{\omega + \mu} I^* \quad (5.4)$$

Substituting equation (5.3) and (5.4) into the first equation in system (3.1), we have

$$I^* = \frac{(\omega + \mu)[\beta\Lambda - \mu(\gamma + \mu)]}{\beta\mu[\omega + \gamma + \mu]} \quad (5.5)$$

Therefore, the threshold $R_01 = \frac{S^0}{S^*} = \frac{\beta\Lambda}{\mu(\gamma+\mu)} > 1$, we have the positive equilibrium,

$$E_1^* = (S_1^*, I_1^*, R_1^*, A_1^*) = (\frac{\gamma + \mu}{\beta}, \frac{(R_01 - 1)(\gamma + \mu)(\omega + \mu)}{\beta[\omega + \gamma + \mu]}, \frac{\gamma}{\omega + \mu} I^*, 0) \quad (5.6)$$

For the stability, the Jacobian of the system is given as

$$J(E_1^*) = \begin{pmatrix} \frac{\partial S_1^*}{\partial S} & \frac{\partial S_1^*}{\partial I} & \frac{\partial S_1^*}{\partial R} & \frac{\partial S_1^*}{\partial A} \\ \frac{\partial I_1^*}{\partial S} & \frac{\partial I_1^*}{\partial I} & \frac{\partial I_1^*}{\partial R} & \frac{\partial I_1^*}{\partial A} \\ \frac{\partial R_1^*}{\partial S} & \frac{\partial R_1^*}{\partial I} & \frac{\partial R_1^*}{\partial R} & \frac{\partial R_1^*}{\partial A} \\ \frac{\partial A_1^*}{\partial S} & \frac{\partial A_1^*}{\partial I} & \frac{\partial A_1^*}{\partial R} & \frac{\partial A_1^*}{\partial A} \end{pmatrix}$$

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Hence, we have

$$|J(E_1^*) - \lambda I| = \begin{bmatrix} -(\beta I_1^* + \mu) - \lambda & -\beta S_1^* & \omega & -\delta S_1^* \\ \beta I_1^* & \beta S_1^* - (\gamma + \mu) - \lambda & 0 & -\theta I_1^* \\ 0 & \gamma & -(\omega + \mu) - \lambda & 0 \\ 0 & 0 & 0 & \theta S_1^* + \theta I_1^* - \mu - \lambda \end{bmatrix}$$

Since $\beta S_1^* - (\gamma + \mu) = 0$, the characteristic equation of $J(E_1^*)$ is given by

$(\delta S_1^* + \theta I_1^* - \mu - \lambda)F(\lambda) = 0$,

where,

$$F(\lambda) = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3,\ a_1 = \beta I_1^* + \mu + \omega + \mu > 0,\ a_2 = (\omega + \mu)(\beta I_1^* + \mu) > 0,\ a_3 = \beta I_1^*[(\omega + \mu)\beta S_1^* - \omega\gamma],$$

Substituting

$$S^* = \frac{\gamma + \mu}{\beta}$$

into a_3, we have

$a_3 = \beta I_1^*[(\omega + \mu)(\gamma + \mu) - \omega\gamma] > 0$.

It is easy to calculate $a_1a_2 - a_3 > 0$. Hence, the eigenvalues of $F(\lambda) = 0$ have negative real parts.

Therefore, according to Routh-hurwitz criterion, the endemic equlibrium point E_1^* is locally stable if $R_{01} > 1$ and $R_{02} - \frac{\delta S_1^* + \theta I_1^*}{\mu} < 1$.

2. When A 6= 0, taking advantage of the second equation of system (3.1), we have

$$A_2^* = \frac{\beta S_2^* - (\gamma + \mu)}{\theta} \tag{5.7}$$

According to the fourth equation of system (3.2), we have

$$I_2^* = \frac{\mu - \delta S_2^*}{\theta} \tag{5.8}$$

$$R_2^* = \frac{\gamma}{(\omega + \mu} I_2^* \tag{5.9}$$

Substituting into the first equation of system (3.1), we have

$$S_2^* = \frac{\Lambda\theta + \omega\gamma\mu/(\omega + \mu)}{(\beta\mu + \theta\mu + \omega\gamma\delta)/(\omega + \mu) - \delta(\gamma + \mu)} \tag{5.10}$$

In order to make $S > 0$, it must satisfy the following conditions:

$$\beta\mu + \theta\mu + \frac{\omega\gamma\delta}{\omega + \mu} - \delta(\gamma + \mu) > 0 \tag{5.11}$$

that is,

$$R_{03} = \frac{(\beta\mu + \theta\mu + \omega\gamma\delta)/(\omega + \mu)}{\delta(\gamma + \mu)} > 1 \tag{5.12}$$

Therefore, if the threshold $R_{03} > 1$, we obtain that

$$S_2^* = \frac{\Lambda\theta + \omega\gamma\mu/(\omega + \mu)}{\delta(\gamma + \mu)(R_{03} - 1)}. \tag{5.13}$$

Hence, we have the positive equlibrium $E_1 = (S^*, I^*, R^*, 0)$ where

$$\begin{cases} S^* = \frac{\gamma + \mu}{\beta} \\ R^* = \frac{\gamma}{\omega + \mu} I^* \\ I^* = \frac{(\omega + \mu)[\beta\Lambda - \mu(\gamma + \mu)]}{\beta\mu[\omega + \gamma + \mu]} \end{cases} \tag{5.14}$$

And the third is the endemic equilibrium point with none of the compartments equals zero, i.e., $E_2 = (S^*, I^*, R^*, A^*)$ where

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$$\begin{cases} A_2^* = \frac{\beta S_2^* - (\gamma+\mu)}{\theta} \\ I_2^* = \frac{\mu - \delta S_2^*}{\theta} \\ R_2^* = \frac{\gamma}{(\omega+\mu)} I_2^* \\ S_2^* = \frac{\Lambda\theta + \frac{\omega\gamma\mu}{\omega+\mu}}{(\beta\mu+\theta\mu+\frac{\omega\gamma\delta}{(\omega+\mu)} - \delta(\gamma+\mu)} \end{cases} \quad (5.15)$$

A sufficient condition for the local asymptotic stability of the equilibrium point is that the eigenvalues, λ_i, of the Jacobian matrix of E_2^* satisfy the condition $|\arg(\lambda_i)| = \frac{\alpha\pi}{2}$. This confirms that fractional-order differential equations are, at least, as stable as their integer order derivative.

6. Numerical Simulation

In this section, the predictor corrector method is applied to get the numerical solutions of system (6) [15].We will propose two cases for the model (3.1) with various of values of parameters. In the first case, $\Lambda = 1, \beta = 0.05, \mu = 0.05, \gamma = 0.96, \omega = 0.8, \delta = 0.00045, \theta = 0.0025$ and with initial conditions: $S(0) = 50, I(0) = 20, R(0) = 1, A(0) = 1$. In this case, $R_0 = 0.9 < 1$, then the virus-free equilibrium is globally stable and the computer virus is eliminated. In the second case, $\Lambda = 1, \beta = 1, \mu = 0.05, \gamma = 0.8, \omega = 0.8, \delta = 0.0045, \theta = 0.0025$ and with initial conditions: $S(0) = 50, I(0) = 20, R(0) = 1, A(0) = 1$. In this case, $R_0 = 23.5 > 1$ which implies that the Computer virus still persists and the viral equilibrium is globally stable.

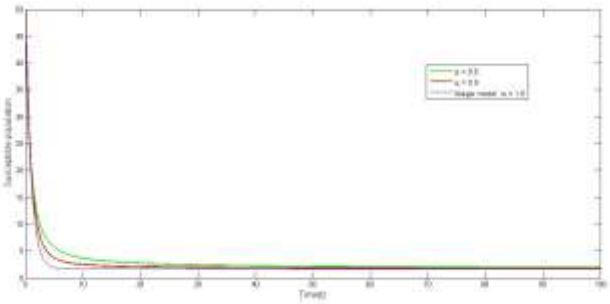

Figure 2: Dynamics of the Susceptible Population.

Figure 2 shows the variation of the susceptible population with time when $R_0 < 1$. This shows that the model would be asymptotically stable when $R_0 < 1$ which means that the virus will not invade the population rather it will die off with time as the decreasing curve does not intercept the horizontal axis. This also shows that the fractional order (like $\alpha = 0.8$) gave a better result than the integer order ($\alpha = 1.0$).

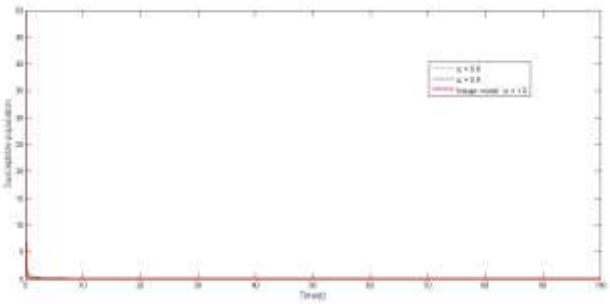

Figure 3: Dynamics of the Suscpetible population with $R_0 > 1$.

Figure 3 shows that when $R_0 > 1$, the susceptible population would continue to decrease until they go into extinction. Hence, there is a need to reduce the contact rate between susceptible and infected computers. It is advised that an antivirus be installed on every computer to avoid the spread.

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Figure 4 shows that recovered computer from the network will increase over time as the infected population is decreasing until no computer is infected when $R_0 < 1$. At this point, we say the system has attained equilibrium. Hence, various methods of avoiding computer viruses should be advised.

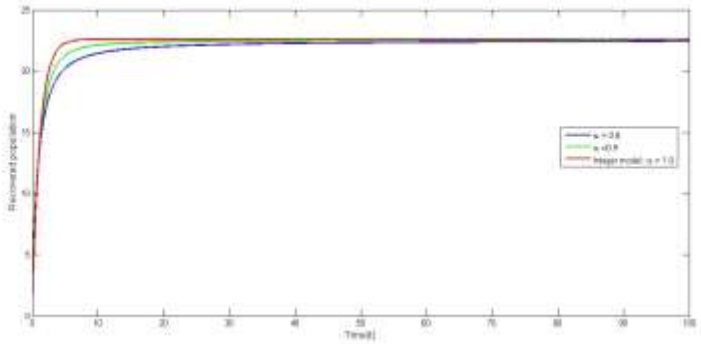

Figure 4: Dynamics of the Recovered Class

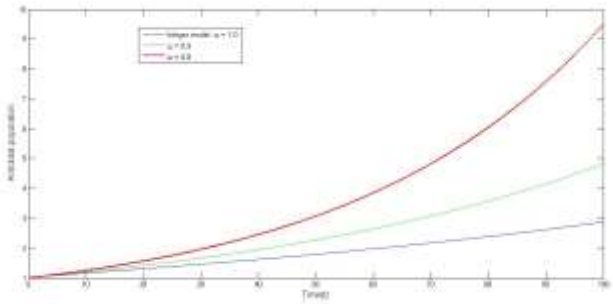

Figure 5: Dynamics of the Antidotal Class

Figure 5 shows that the Antidotal class increases over time when $R_0 < 1$ thereby making the model stable. Clearly, it can also be seen that as the value of α increases, the model becomes unstable telling us that the fractional case gives a better result than the classical case.

7. Conclusion

In this paper,the fractional order model was introduced into a compartmental epidemic model with the incorporation of new computers to the network, the removal of old computers from the network, the computer equipped with antivirus software, and so forth. The result shows that the solution continuously depends on the time-fractional derivative. When $\alpha \rightarrow 1$, the solution of the fractional model reduces to the standard solution of the integer order model. The result also shows that the fractional order model gives a better result than the integer model.

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