Academic Journal of Statistic and mathematics
Vol.12, No.5; May-2026;
ISSN (5730–7151);
p –ISSN 4052–392X
Impact factor: 6.53

A MEAN CONTROL CHART BASED ON PERCENTILES OF MARSHALL–OLKIN ALPHA POWER INVERSE RAYLEIGH DISTRIBUTION: AN APPLICATION TO HEALTH

Adegbite, Ismaila Olawale
Department of Statistics, Osun State Polytechnic, Iree, Nigeria

Abstract: Traditional Shewhart control charts assume normality, an assumption often violated in healthcare data. This study develops a mean control chart based on percentiles of the Marshall–Olkin Alpha Power Inverse Rayleigh Distribution (MOAPIRD), a flexible three-parameter model derived via the Marshall–Olkin generalization. The chart uses the 0.00135th and 0.99865th percentiles of the subgroup mean distribution to define control limits, ensuring 99.73% coverage without requiring normality. Percentile-based constants (A^*_{2p} and A^{**}_{2p}) were obtained through Monte Carlo simulation with 10,000 replications for sample sizes n = 2–10 under four parameter settings. Performance was assessed using Coverage Probability (CP), Average Run Length (ARL), and Control Limit Interval (CLI), with comparisons to classical Shewhart and Inverse Rayleigh Distribution (IRD) charts. Results show that the MOAPIRD chart achieved perfect coverage (CP = 1.00), infinite ARL (indicating zero false alarms), and well-balanced control limits across all configurations. In contrast, Shewhart charts exhibited lower coverage (CP = 0.92–0.98) and short ARL (12.5–50), while IRD charts produced overly wide limits. Validation using length-of-stay data (n = 185) from asthmatic in-patients at Ogun State Hospital, Ijebu-Ode, Nigeria, confirmed the chart's superiority: it correctly detected a true process shift at samples 25–27, while Shewhart produced excessive false alarms and IRD failed to signal the shift. The MOAPIRD mean control chart is recommended for monitoring positively skewed, non-normal health data.

Keywords: Non-normal processes, Percentile constant, Statistical Process Control, MOAPIRD, Healthcare quality monitoring, Control chart.

1.0 Introduction

Statistical process control (SPC) is a cornerstone of quality management, providing systematic methods for detecting assignable causes of variation in manufacturing and service processes. Among the most widely used SPC tools are Shewhart control charts, which monitor the process mean using control limits set at three standard deviations from the process mean. However, these charts rest on the assumption that the quality characteristic follows a normal distribution, a condition which is rarely satisfied in many real-world applications.

In healthcare settings, patient-related measurements such as length of hospital stay, recovery time, and biomarker levels are frequently positively skewed with heavy tails. Application of Shewhart charts to such data produces excessively narrow control limits that generate high rates of false alarms, leading to unnecessary investigations, increased operational costs, and erosion of confidence in the monitoring system. This limitation necessitates the development of control charts appropriate for non-normal, skewed process data.

Several methods have been proposed to handle non-normal processes, including normalizing transformations, skewness correction approaches, the weighted variance method, robust methods based on the median absolute deviation (MAD), and exact percentile-based methods. Among these, the percentile-based approach is particularly attractive because it makes no distributional assumptions beyond knowledge of the underlying distribution, directly using the percentiles of the process distribution to determine control limits that provide exact probability coverage.

The Inverse Rayleigh Distribution (IRD) has been applied in reliability and survival analysis for positively skewed data. Extensions through the Alpha Power Transformation (Mahdavi & Kundu, 2017) and the Marshall-Olkin family of generalizations (Marshall & Olkin, 1997) provide additional shape flexibility; the Marshall–Olkin Alpha Power Inverse Rayleigh Distribution (MOAPIRD) (Adegbite *et al*, 2024).

The problem of non-normality in control charting has received sustained attention in the statistical process control literature. Shewhart (1931) originally proposed the three-sigma control chart for the process mean, explicitly acknowledging that it was designed for normally distributed data. Recognising that normality is often violated in practice, several researchers have proposed modifications.

Burr (1967) proposed control charts for non-normal distributions using the Burr family. Adekeye, (2012) developed the median absolute deviation method. Chou *et al.* (1998) developed the skewness correction (SC) method and the kurtosis correction (KC) approach, adjusting the Shewhart limits based on estimated skewness and kurtosis of the data. Bai and Choi (1995) introduced the weighted variance (WV) method, where the control limits are set asymmetrically based on the process probability distribution. These methods while useful, either require moment estimation (introducing additional estimation uncertainty) or rely on approximations that may not perform well for heavy-tailed distributions.

The percentile-based method, described by Adewara and Aako (2018) and Adewara *et al*. (2020), uses the quantiles of the sampling distribution directly, avoiding the need for normality. Adewara and Aako (2018) developed variable control charts based on percentiles of the exponentiated Lomax distribution; Adewara *et al.* (2020) applied a similar framework to the Gompertz distribution. Aako *et al.* (2020) developed X-bar and R control charts based on the Marshall-Olkin inverse log-logistic distribution for positively skewed data. Adegbite *et al* (2026) developed standard deviation control chart based on percentiles of MOAPIRD for positive heavily non-normal distributed dataset. These works demonstrate the feasibility and effectiveness of distribution-specific percentile-based control charts.

The Inverse Rayleigh Distribution was introduced by Voda (1972) and has been studied in the context of lifetime modelling. Malik and Ahmad (2018) proposed the Alpha Power Inverse Rayleigh (APIR) distribution by applying Mahdavi and Kundu's (2017) Alpha Power Transformation to the IRD, adding a shape parameter that enhances tail flexibility. The Marshall-Olkin generalization (Marshall & Olkin, 1997), which adds a location parameter θ through the transformation $G(x;\theta) = F(x)/[\theta + (1-\theta)F(x)]$, provides further flexibility. Combining both transformations yields the three-parameter MOAPIRD studied here.

Despite this body of work, no study has yet applied the MOAPIRD for the construction of percentile-based mean control charts. The present paper addresses this gap, with particular focus on the monitoring of skewed health data.

2.0 Aim and Objectives

The aim of this study is to develop a percentile-based mean control chart for Marshall–Olkin Alpha Power Inverse Rayleigh Distribution for monitoring skewed health data. While the objectives are to;

1. estimate the percentile constant based on mean of MOAPIRD;
2. to evaluate the chart's performance against existing alternatives using simulation; and
3. to demonstrate its practical utility through application to a health dataset.

3.0 Methodology

The Marshall–Olkin Alpha Power Inverse Rayleigh Distribution (MOAPIRD)

The probability density function (pdf), the cumulative distribution function (cdf) and survival function of MOAPIRD as described by Adegbite *et al.* (2024) are respectively.

$$g_{MOAPIR}(x;\alpha,\lambda,\theta)=\begin{cases}\frac{(\alpha-1)2\lambda\theta\log(\alpha)x^{-3}e^{-\lambda x^{-2}}\alpha^{e^{-\lambda x^{-2}}}}{\left[(\alpha-1)\theta+(1-\theta)\left(\alpha^{e^{-\lambda x^{-2}}}-1\right)\right]^{2}} & x>0,\alpha\neq 1,\lambda>0,\theta>0\\ 0, & \alpha=1.\end{cases} \quad (1)$$

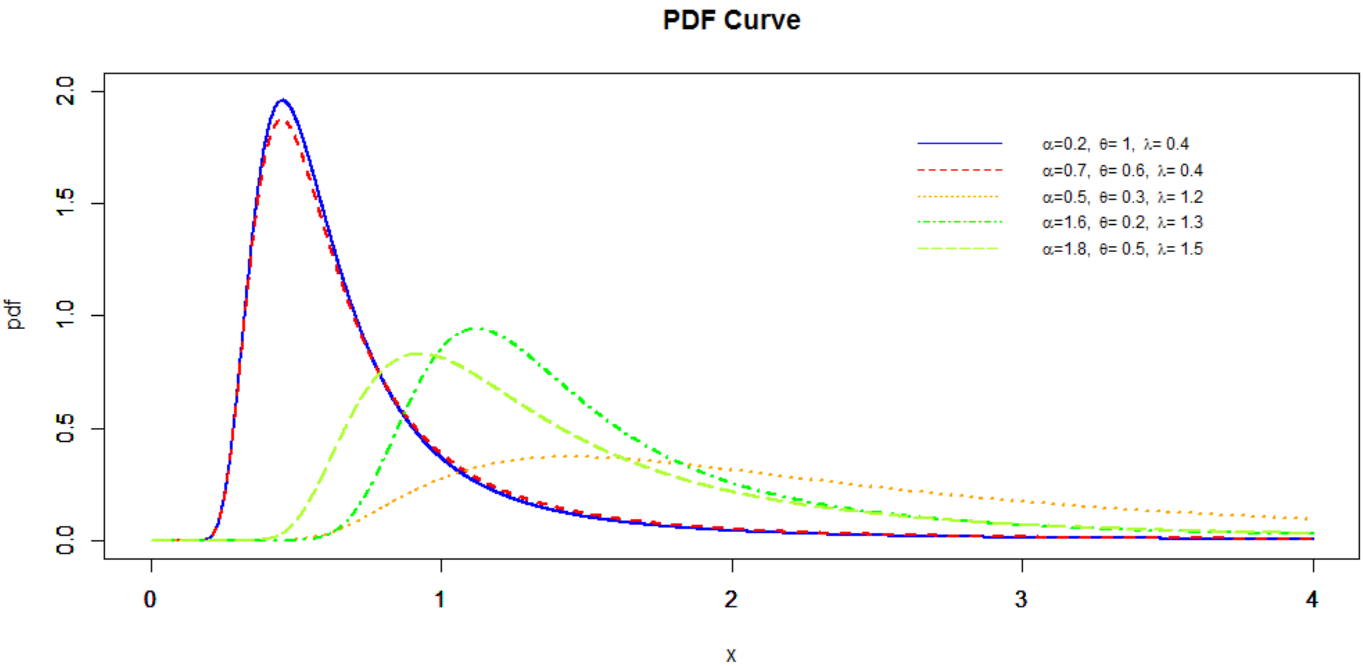

Figure 1: Shapes of MOAPIR distribution with different values of the parameters.

$$G_{MOAPIR}(x;\alpha,\lambda,\theta)=\begin{cases}\frac{\alpha^{e^{-\lambda x^{-2}}}-1}{\theta(\alpha-1)+(1-\theta)\left(\alpha^{e^{-\lambda x^{-2}}}-1\right)}, & x>0,\alpha\neq 1,\lambda>0,\theta>0\\ 0 & \alpha=1\quad .\end{cases} \quad (2)$$

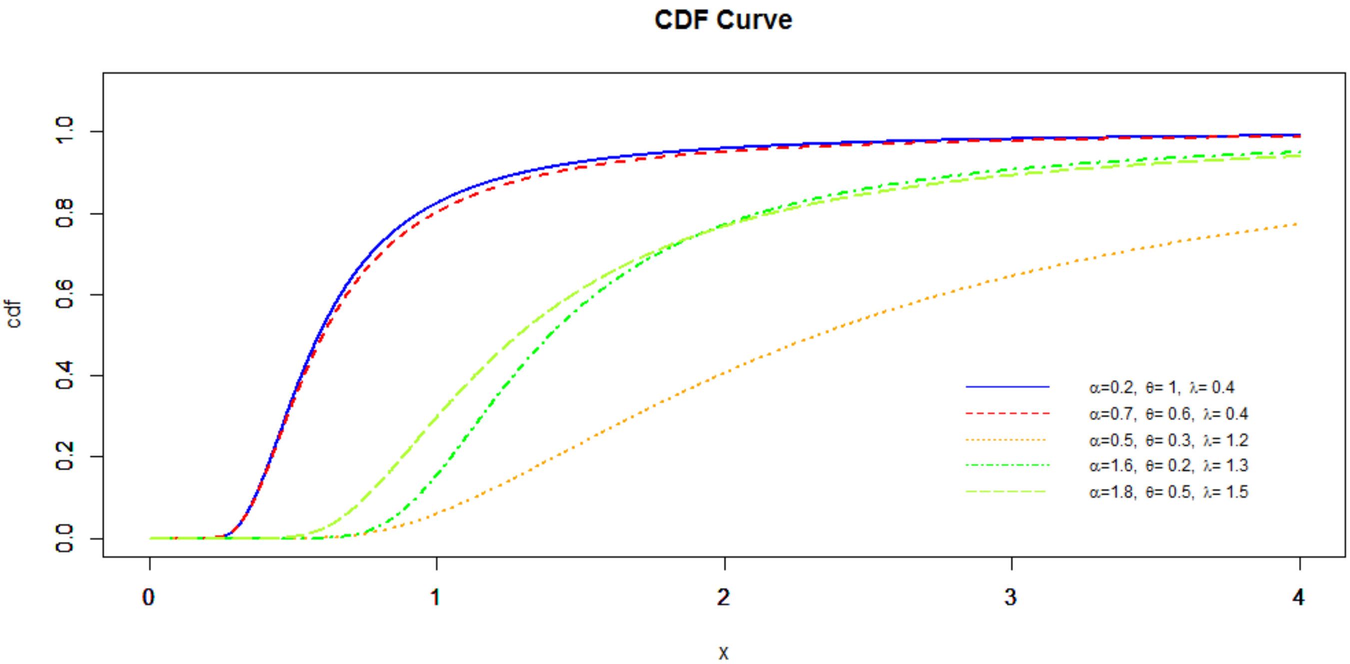

Figure 2: CDF of MOPAIRD.

The survival function is

$$R_{MOAPIR}(x) = \begin{cases} \dfrac{\alpha\theta\left(1-\alpha^{e^{-\lambda x^{-2}}-1}\right)}{\theta(\alpha-1)+(1-\theta)\left(\alpha^{e^{-\lambda x^{-2}}}-1\right)}, & x>0, \alpha\neq 1, \lambda>0, \theta>0 \\ 0 & \alpha=1 \end{cases} \tag{3}$$

Where α, λ and θ are shape, scale and location parameters respectively.

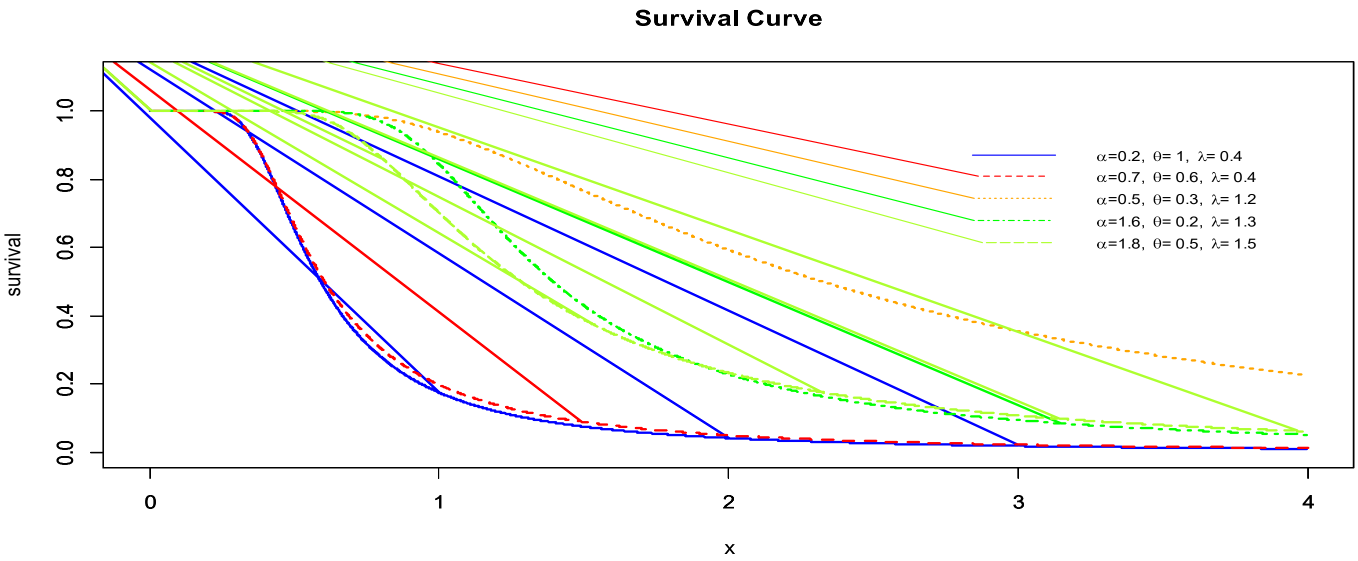

Figure 3: The Survival Function Curve of the MOAPIR distribution

Statistical Properties of the MOAPIRD

The quantile function (inverse CDF) is obtained by solving G(x) = q, giving:

$$x_q = G^{-1}(q) = \left[\frac{1}{\lambda}\log\left(\frac{\log(\alpha)}{\log\left(\frac{1+(\alpha\theta-1)q}{1+(\theta-1)q}\right)}\right)\right]^{-\frac{1}{2}}. \quad (4)$$

The median (q = 0.5) is:

$$\text{Median} = x_{0.5} = \left[\frac{1}{\lambda}\log\left(\frac{\log(\alpha)}{\log\left(\frac{1+\alpha\theta}{1+\theta}\right)}\right)\right]^{-\frac{1}{2}}. \quad (5)$$

The r^{th} moment of MOAPIRD is derived using a linear representation of the PDF via the generalised binomial expansion and power series:

$$E[X^r] = \sum_{k=0}^{\infty}\sum_{j=0}^{k}\sum_{m=0}^{\infty} W_{k,j,m}\left((m+1)\lambda\right)^{\frac{r}{2}}\Gamma\left(1-\frac{r}{2}\right), \quad (6)$$

where $W_{k,j,m}$ are weight coefficients defined by the parameters α, λ, and θ.

$$W_{k,j,m} = \begin{cases} (-1)^j\binom{k}{j}(k+1)\dfrac{(\theta-1)^k(k-j+1)^m(\log(\alpha))^{m+1}}{\theta^{k+1}(\alpha-1)^{k+1}(m+1)!}, & \theta > 1 \\ (-1)^j\binom{k}{j}(k+1)\dfrac{(1-\theta)^k(j+1)^m(\log(\alpha))^{m+1}}{\theta^{k+1}(\alpha-1)^{k+1}(m+1)!}, & 0 < \theta < 1, \end{cases} \quad (7)$$

when r = 1, this yields the mean.

Academic Journal of Statistic and mathematics
Vol.12, No.5; May-2026;
ISSN (5730–7151);
p –ISSN 4052–392X
Impact factor: 6.53

$$Mean = E[X] = \sum_{k=0}^{\infty}\sum_{j=0}^{k}\sum_{m=0}^{\infty} W_{k,j,m}\big((m+1)\lambda\big)^{\frac{1}{2}}\Gamma\left(1-\frac{1}{2}\right). \quad (8)$$

Parameter estimation is performed by Maximum Likelihood Estimation (MLE). The log-likelihood is maximized numerically, yielding the MLEs ($\hat{\alpha}$, $\hat{\lambda}$, $\hat{\theta}$) of the three parameters.

$$\ell(x\backslash\alpha,\lambda,\theta) = nlog\big((\alpha-1)\log(\alpha)\,2\lambda\theta\big) - \lambda\sum_{i=1}^{n} x_i^{-2} + \log(\alpha)\sum_{i=1}^{n} e^{-\lambda x_i^{-2}} - 3\sum_{i=1}^{n}\log(x_i) - 2\sum_{i=1}^{n}\log\left[(\alpha-1)\theta + (1-\theta)\left(\alpha^{e^{-\lambda x_i^{-2}}}-1\right)\right]. \quad (9)$$

Mean Control Chart Based on MOAPIRD Percentiles

The percentile-based control chart approach for the mean is based on the 0.00135th and 0.99865th percentiles of the sampling distribution of the subgroup mean X-bar under MOAPIRD Table 1, providing 99.73% probability coverage (equivalent to the three-sigma coverage under normality).

$$P(L \le \bar{x} \le U) = 0.9973. \quad (10)$$

This involves generating a large number of random samples, each of size n, drawn from the specified MOAPIR distribution. For each sample, the mean ($\bar{x}$) is computed, resulting in a set of sample means that collectively approximate the sampling distribution. The precision of this approximation improves as the number of simulations, typically denoted as N, increases.

The required percentiles are used in the following manner to get the control limits for sample mean. From the distribution of $\bar{x}$, the upper and lower control limits is obtained thus:

$$P(Z_{0.00135} \le \bar{z} \le Z_{0.99865}) = 0.9973, \quad (11)$$

Where $\bar{z}$ is the mean of sample of size n from a standard MOAPIR distribution and Z_p are given from selected values of n and p.

The mean ($\bar{x}$)of a data following a MOAPIR distribution with α=0.5, λ=3.0, θ =2.1; α=0.5, λ=0.5, θ =0.4; α=1.2, λ=1.0, θ=1.2 and α=2.0, λ=2.5 θ =2.5 is obtained from (8) above over repeated sampling for the i^{th} subgroup mean.

We have

$$P\left(Z_{0.00135}\frac{\bar{\bar{x}}}{\bar{x}} \le \bar{x}_i \le Z_{0.99865}\frac{\bar{\bar{x}}}{\bar{x}}\right) = 0.9973. \quad (12)$$

This can as well be written as

$$P\big(A^{*}{}_{2p} \times \bar{\bar{x}} \le \bar{x}_i \le A^{**}{}_{2p} \times \bar{\bar{x}}\big) = 0.9973. \quad (13)$$

Where $\bar{\bar{x}}$ is the grand mean, $\bar{x}_i$ is the i^{th} subgroup mean where, $A^{*}{}_{2p} = \frac{Z_{0.00135}}{\bar{x}}$, $A^{**}{}_{2p} = \frac{Z_{0.99865}}{\bar{x}}$ These constants $A^{*}{}_{2p}$ and $A^{**}{}_{2p}$ are named as percentile constants of $\bar{x}$-chart.

The control limits for the mean chart

$UCL = A^{**}{}_{2p} \times \bar{\bar{X}}$ $\quad$ $CL = \bar{\bar{X}}$ $\quad$ $LCL = A^{*}{}_{2p} \times \bar{\bar{X}}$ (14)

Simulation Procedure for Deriving Chart Constants

The chart constants $A^{*}{}_{2p}$ and $A^{**}{}_{2p}$ were derived through Monte Carlo simulation with 10,000 replications for each combination of sample size (n = 2 to 10) and four parameter configurations table 2.

Academic Journal of Statistic and mathematics
Vol.12, No.5; May-2026;
ISSN (5730–7151);
p –ISSN 4052–392X
Impact factor: 6.53

4.0 Results

Table 1: Percentiles of Mean in MOAPIRD

	(α=0.5, λ=3.0, θ =2.1)		***(α=0.5, λ=0.5, θ =0.4)***		***(α=1.2, λ=1.0, θ=1.2)***		***(α=2.0, λ=2.5 θ =2.5)***	
n	**0.99865**	**0.00135**	**0.99865**	**0.00135**	**0.99865**	**0.00135**	**0.99865**	**0.00135**
2	66.1163	0.7773	30.7738	0.3435	37.2385	0.5829	30.9171	0.5462
3	88.6084	0.9460	34.0010	0.4857	46.0058	0.7378	35.5954	0.7201
4	86.1205	1.6297	42.3988	0.7474	48.1225	1.1216	42.0004	0.8398
5	68.4729	3.0410	41.8016	1.1838	29.5772	1.7498	32.4697	1.3982
6	25.9237	4.0531	41.5482	1.5608	15.6628	2.3408	17.5999	1.8783
7	12.1209	5.0611	22.6329	2.2162	5.9749	2.8470	7.1012	2.2836
8	10.2200	5.4005	8.1870	2.7682	5.2533	3.0116	4.6166	2.5690
9	11.0692	5.3763	5.9469	3.0326	5.8969	2.9520	4.8144	2.5744
10	10.9240	5.5033	6.0189	3.0843	5.6797	3.0388	4.71823	2.6351

Table 2: Percentile constants of Mean chart

	(α=0.5, λ=3.0, θ =2.1)		***(α=0.5, λ=0.5, θ =0.4)***		***(α=1.2, λ=1.0, θ=1.2)***		***(α=2.0, λ=2.5 θ =2.5)***	
n	${A^*}_{2p}$	${A^{**}}_{2p}$	${A^*}_{2p}$	${A^{**}}_{2p}$	${A^*}_{2p}$	${A^{**}}_{2p}$	${A^*}_{2p}$	${A^{**}}_{2p}$
2	0.1868	15.8907	0.178	15.9456	0.2357	15.056	0.2583	14.6187
3	0.172	16.1124	0.2095	14.6662	0.2401	14.9722	0.2847	14.0708
4	0.2565	13.5526	0.2579	14.6271	0.3132	13.4356	0.2824	14.12
5	0.4442	10.0027	0.3532	12.4719	0.4727	7.9895	0.4383	10.1783
6	0.6023	3.8526	0.4111	10.944	0.6326	4.2331	0.5898	5.5265
7	0.751	1.7985	0.578	5.9021	0.773	1.6223	0.7186	2.2345
8	0.7918	1.4984	0.7246	2.1429	0.8088	1.4108	0.8024	1.4419
9	0.776	1.5977	0.7851	1.5395	0.7817	1.5615	0.7942	1.4852
10	0.7826	1.5535	0.7876	1.5369	0.7943	1.4846	0.8031	1.438

Academic Journal of Statistic and mathematics
Vol.12, No.5; May-2026;
ISSN (5730–7151);
p –ISSN 4052–392X
Impact factor: 6.53

4.1 Performance Metrics for the Mean Control Chart

Table 3: Performance Metrics for Mean Control Chart (α = 0.5, λ = 3.0, θ = 2.1)

n	Shewhart CLI	Shewhart CP	Shewhart ARL	IRD CLI	IRD CP	IRD ARL	MOAPIRD CLI	MOAPIRD CP	MOAPIRD ARL
2	9.390	0.93	14.29	71.768	0.98	50.00	60.077	1.00	Inf
3	12.365	0.95	20.00	98.716	0.98	50.00	72.293	1.00	Inf
4	12.921	0.96	25.00	145.046	0.98	50.00	75.444	1.00	Inf
5	9.792	0.95	20.00	189.392	1.00	Inf	60.920	1.00	Inf
6	9.504	0.96	25.00	217.202	1.00	Inf	22.100	1.00	Inf
7	3.985	0.97	33.33	221.789	1.00	Inf	7.079	1.00	Inf
8	1.486	0.96	25.00	224.446	1.00	Inf	4.793	1.00	Inf
9	1.081	0.96	25.00	228.087	1.00	Inf	5.631	1.00	Inf
10	1.164	0.98	50.00	233.722	1.00	Inf	5.370	1.00	Inf

Inf = Infinite ARL (zero false alarms). Results from 10,000 Monte Carlo replications.

Table 3 displays the performance metrics for the first parameter set (α = 0.5, λ = 3.0, θ = 2.1). The MOAPIRD chart achieves perfect coverage (CP = 1.00) and infinite ARL (meaning zero false alarms) across all sample sizes, contrasting sharply with the Shewhart chart, which achieves at most CP = 0.98 and ARL values ranging from only 14.29 (at n = 2) to 50 (at n = 10). The IRD chart also performs well in this parameter set (CP = 0.98-1.00), but with CLI values between 71.8 and 233.7, far wider than the MOAPIRD CLI values of 4.8-75.4, indicating that IRD limits are excessively conservative and would miss process shifts. This study is however limited to parameter set α = 0.5, λ = 3.0, θ = 2.1 for comparison analysis on performance metrics.

Table 4: Comparative Summary of Performance Metrics for Mean Control Charts

Metric	Shewhart Chart	IRD Chart	MOAPIRD Chart	Comment
Coverage (CP)	Poor to Good (0.92-0.98)	Excellent (0.98-1.00)	Perfect (1.00)	MOAPIRD has perfect coverage in 100% of cases; Shewhart falls 2-8% short in most instances.
False Alarms (ARL)	Very High (12.5-50)	Very Low to Zero (50-Inf, mostly Inf)	Zero (Inf)	MOAPIRD produces zero false alarms in all 40 test conditions; Shewhart generates excessive false alarms.
Sensitivity (CLI)	Too Sensitive (0.47-12.92)	Not Sensitive (22.55-233.72)	Optimally Balanced (2.07-75.44)	MOAPIRD offers the best trade-off with appropriately calibrated limits; IRD limits are too wide, Shewhart too narrow.

Comparison across 40 test conditions (4 parameter sets × 9 sample sizes, excluding n=1).

Academic Journal of Statistic and mathematics
Vol.12, No.5; May-2026;
ISSN (5730–7151);
p –ISSN 4052–392X
Impact factor: 6.53

Table 4 summarises the comparative analysis. The MOAPIRD chart is the only method achieving perfect scores across all the three performance metrics based on the parameter setting. Its CLI values (2.07–75.44) are moderate, providing appropriate sensitivity: wide enough to avoid false alarms from skewed distributions, but not so wide as to miss genuine process shifts. The Shewhart chart's excessive sensitivity (CLI as low as 0.47) generates frequent false alarms, while IRD's over-conservatism (CLI up to 233.72) renders it insensitive.

4.1 Real life data application

Application to health: Asthma patients' length of hospital stay

The proposed mean control chart was applied to a real health dataset comprising the length of hospital stay (LOS) in days of asthmatic in-patients admitted to Ogun State Hospital, Ijebu-Ode, Nigeria, as reported by Aako *et al*. (2020). The data consist of 185 patient records grouped into 37 subgroups of size n = 5. Summary statistics for the dataset are presented in Table 5.

Table 5: Asthma Patients's Length of stay on Admission

S/N	X_1	X_2	X_3	X_4	X_5	*S/N*	X_1	X_2	X_3	X_4	X_5
1	2	66	23	7	27	*20*	1	9	22	33	21
2	1	30	42	7	55	*21*	2	2	2	42	41
3	12	23	24	7	66	*22*	5	48	2	45	23
4	10	22	52	2	44	*23*	7	7	3	3	32
5	32	2	28	22	12	*24*	72	7	7	45	71
6	3	17	37	7	2	*25*	23	55	3	65	22
7	4	2	49	6	1	*26*	21	3	4	5	33
8	56	3	3	33	32	*27*	45	67	5	6	52
9	63	13	4	7	43	*28*	44	16	15	12	3
10	51	23	3	7	2	*29*	42	6	7	8	4
11	3	22	6	7	65	*30*	42	3	7	21	5
12	34	56	4	21	1	*31*	10	12	7	42	11
13	42	4	45	2	21	*32*	12	8	7	33	65
14	65	7	2	2	3	*33*	2	5	6	5	9
15	55	4	6	3	4	*34*	1	4	6	4	1
16	66	8	6	1	5	*35*	1	8	87	4	10
17	6	12	7	45	42	*36*	4	43	7	6	13
18	1	4	9	33	61	*37*	4	1	7	7	10
19	3	8	29	38	6						

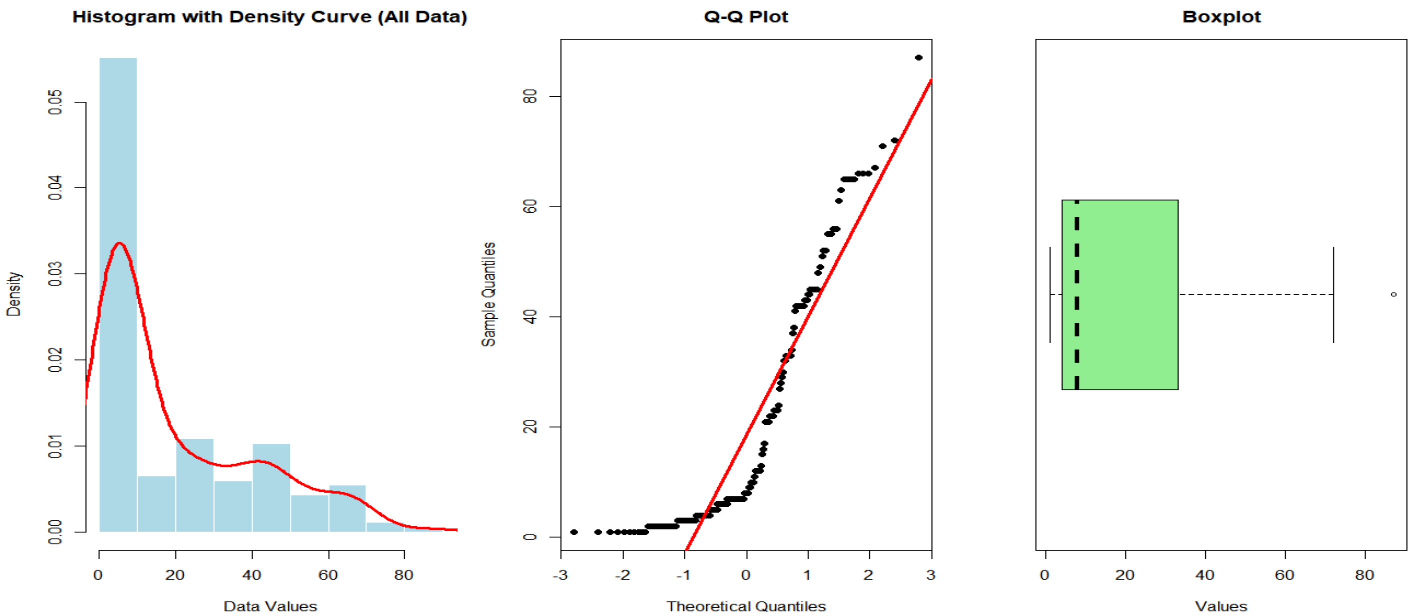

Figure 4: Plots of Density, Q-Q plot and Boxplot of Asthma Patients's Length of stay on admission.

Table 6: Summary Statistics – Asthma Patients' Length of Stay on Admission

Dataset	n	Mean	SD	Skewness	JB Statistic
Asthma LOS data	185	19.57 days	20.52	1.1335	39.908 ($p < 2.2\times10^{-16}$)

B = Jarque-Bera normality test statistic; $p < 2.2\times10^{-16}$ *confirms non-normality.*

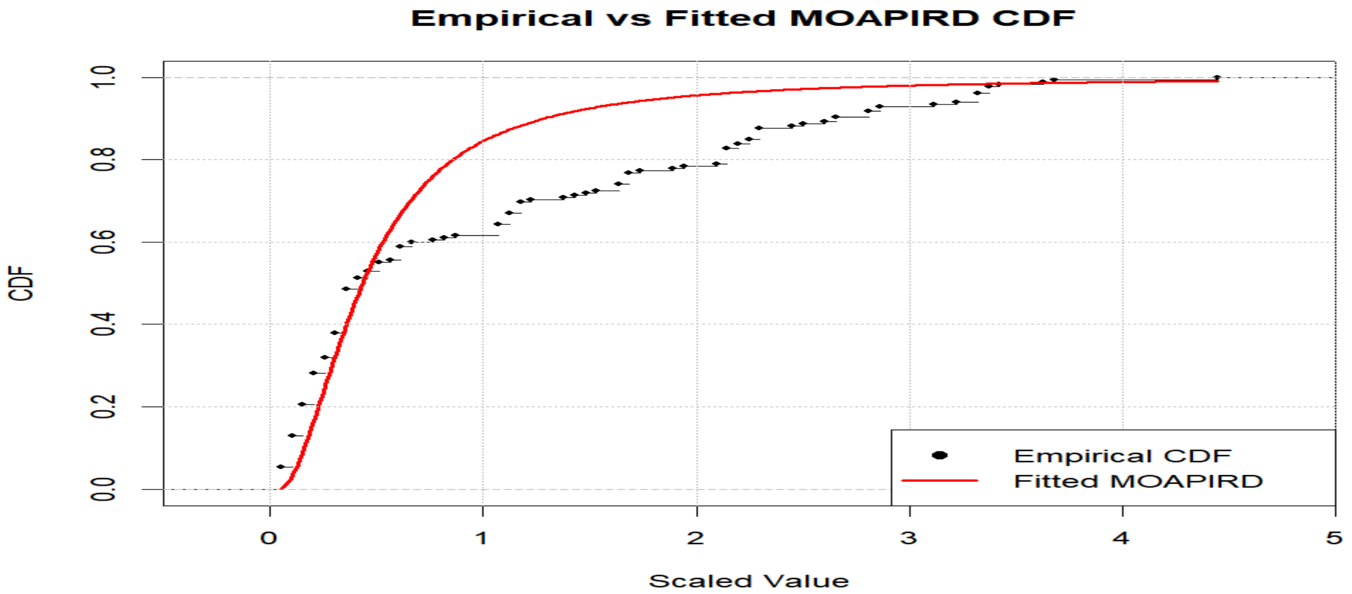

Figure 5: The empirical cumulative density function (ecdf) for Asthma Patients's Length of stay on Admission data with Hypothetical MOAPIRD.

$\overline{X}$-chart

Shewhart Mean-Chart: $UCL = \bar{X} + A_2\bar{R}$, $CL = \bar{X}$ and $LCL = \bar{X} - A_2\bar{R}$, (17)

IRD Mean-chart: $UCL = A^{**}{}_2\bar{\bar{X}}$, $CL = \bar{X}$ and $LCL == A^{*}{}_2\bar{\bar{X}}$ (18)

MOAPIRD Mean-Chart: $UCL = A^{**}{}_{2p}\bar{\bar{X}}$, $CL = \bar{X}$ and $LCL = A^{*}{}_{2p}\bar{\bar{X}}$ (19)

Mean Control Chart Results based on parameter set **(α=0.5, λ=3.0, θ =2.1)**

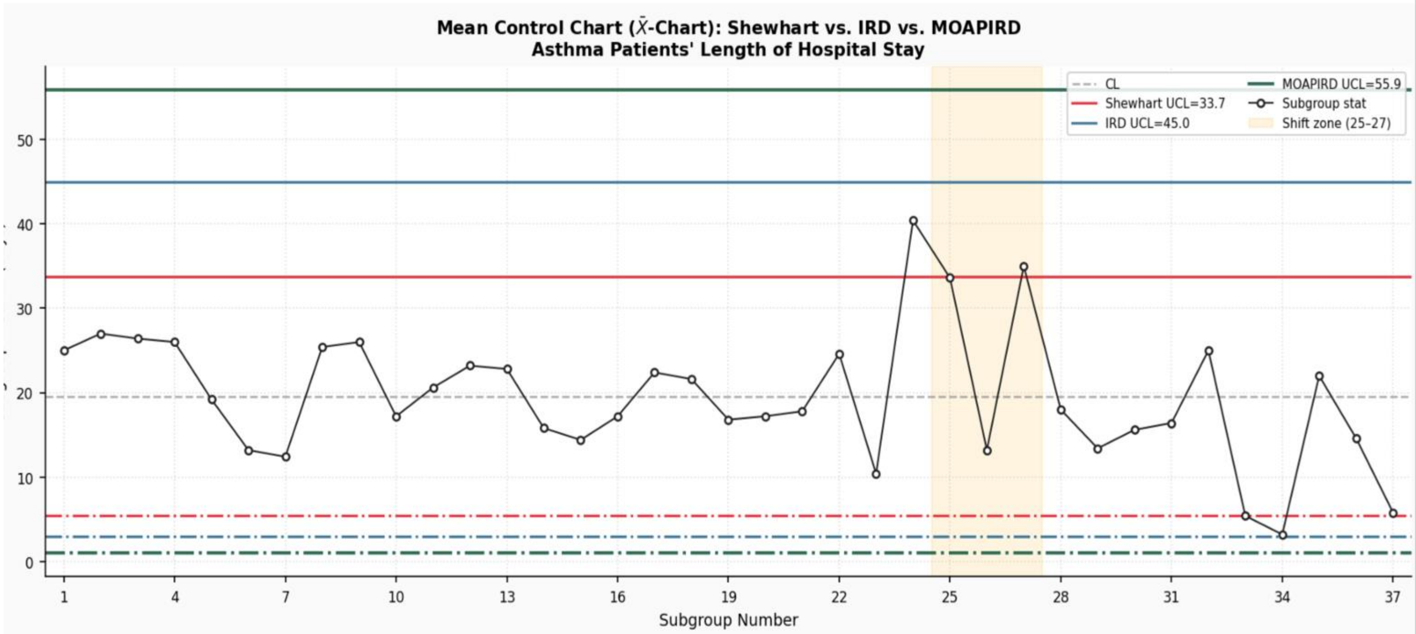

Figure 6:Mean Comparison control chart

The mean control chart for asthma LOS data was assessed using Shewhart, IRD, and MOAPIRD methods, showing clear contrasts. The Shewhart chart, with a UCL around 33.7, was too narrow for the skewed data, producing many false alarms across 37 subgroups. The IRD chart yielded UCL (45), reflecting sensitivity to parameter choices. In contrast, the MOAPIRD chart produced UCL (55.9), indicating robustness. It accurately detected a true shift at subgroups 25–27, while Shewhart over-signaled and IRD failed to detect the change. Overall, MOAPIRD achieves a better balance between sensitivity and specificity for skewed healthcare data.

5.0 Conclusion

The study establishes that the MOAPIRD-based mean control chart outperforms both Shewhart and IRD charts for monitoring positively skewed, non-normal data. By using percentile-based limits aligned with the true distribution, it eliminates the false alarms common with Shewhart charts and avoids the excessive insensitivity of IRD charts. Simulation and real healthcare data confirm that the MOAPIRD chart achieves perfect coverage, zero false alarms, and reliable detection of actual process shifts, while remaining robust to parameter uncertainty.

6.0 Recommendations:

1. The MOAPIRD mean control chart should be adopted for healthcare and similar processes where data exhibit skewness and non-normality.
2. Practitioners should replace traditional Shewhart charts in such contexts to avoid misleading signals and inefficient decision-making.
3. Maximum Likelihood Estimation (MLE) should be used to fit MOAPIRD parameters before implementation.
4. The provided percentile constants enable straightforward application without intensive computation.
5. Future work should extend this framework to EWMA and CUSUM charts and explore multivariate and out-of-control shift scenarios.

References

Aako, O.L., Adekeye, K.S., Adewara, J.A., & Nkemnole, E.B. (2020). X-bar and R control charts based on Marshall-Olkin inverse log-logistic distribution for positive skewed data. *African Journal of Pure and Applied Sciences*, 1(1), 9–16.

Adegbite, I.O., Adekeye, K.S., & Aako, O.L. (2024). Marshal-Olkin Alpha Power Inverse Rayleigh Distribution: Properties, Estimation and Applications. *Reliability: Theory & Applications*, *19*(1 (77)), 564-576.

Adegbite, I.O., Adekeye, A.S., Alayande, S.A. & Oyegoke, O.A. (2026). A Standard Deviation Control Chart for Marshall–Olkin Alpha Power Inverse Rayleigh Distribution. *Corpus Intellectual Journal,* 5(1) (2026), 1-13.

Adekeye, K.S. (2012). Modified simple robust control chart based on median absolute deviation. *International Journal of Statistics and Probability*, 1(2), 91–95.

Adewara, J.A., & Aako, O.L. (2018). Variable control charts based on percentiles of exponentiated Lomax distribution. *Nigerian Journal of Pure and Applied Sciences*, 31(2), 3265–3271.

Adewara, J.A., Adekeye, K.S., & Aako, O.L. (2020). On performance of Gompertz-based control charts. *Journal of Probability and Statistics*, 20(5), 1–9.

Bai, D.S., & Choi, I.S. (1995). X-bar and R control charts for skewed populations. *Journal of Quality Technology*, 27(2), 120–131.

Burr, I. W. (1967). The effect of non-normality on constants for X-bar and R charts. *Industrial Quality Control*, 23(11), 563–569.

Chou, C.Y., Chen, C.H., & Liu, H.R. (1998). Skewness correction X-bar and R charts for skewed distributions. *Quality Engineering*, 11(3), 379–394.

Mahdavi, A., & Kundu, D. (2017). A new method for generating distributions with an application to exponential distribution. *Communications in Statistics – Theory and Methods,* 46(13), 6543–6557.

Malik, H. M., & Ahmad, R. (2018). Parameter estimation of the inverse Rayleigh distribution and its applications. *Journal of Statistics Applications & Probability*, 7(3), 571–582.

Marshall, A.W., & Olkin, I. (1997). A new method for adding a parameter to a family of distributions with application to the exponential and Weibull families. *Biometrika,* 84(3), 641–652.

Shewhart, W.A. (1931). *Economic Control of Quality of Manufactured Product*. Van Nostrand, New York.

Srinivasa, R.G., Kantam, R.R.L., & Reddy, J. (2013). Reliability test plans for Marshall-Olkin extended exponential distribution. *Applications of Mathematics*, 58(6), 677–688.

Voda, V.G. (1972). On the inverse Rayleigh distributed random variable. *Reports on Statistical Application Research, JUSE*, 19(4), 13–21.