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MODELING THE FATALITY OF COVID-19 PANDEMIC IN NIGERIA USING SOME REGRESSION MODELS

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Abstract: The outbreak of corona virus (covid-19) pandemic stirs up a lot of responses from researchers all over the world. Statisticians have also deployed useful statistical tools and techniques to provide useful information on the spread of the pandemic to enable state actors respond accordingly. This paper models the fatality of covid-19 pandemic in Nigeria. The data was retrieved from the database of Our World in Data (OWID). A scatter plot of the data revealed an approximately linear relationship between the cumulative total confirmed cases and the cumulative total covid-19 related deaths in Nigeria. Linear regression models involving the logarithmic transformation of the variables were employed in this study. The models include the simple linear, the linear-log, the log-linear, and the log-log regression models. The performances of the various models were compared using the coefficient of determination (R^2). The result showed that, though the studied models were significant at a 0.05 level, the log-log regression model with a coefficient of determination of 0.9780 provides the best fit and hence a better prediction power to the data.

Keywords: COVID-19 pandemic, model, linear regression, log-log regression, coefficient of determination.

1.0 Introduction

The impact of the COVID-19 has an unprecedented effect in global history. Although technological advancements have helped a great deal in its containment, the spread of the pandemic is still a strong challenge. To contain the spread of COVID-19, countries of the world have, at different times pulled resources together, financial, intellectual, and otherwise, to fight what they all perceived to be a common enemy. Despite the effort, there still appears to be a rise in the globally spread with some countries still recording daily new cases and deaths of about one thousand and more. According to reports, the countries most hit by this pandemic are in Europe and North America (World Health Organisation, 2020).

The first confirmed case of COVID-19 case emerged in the month of December, 2019 in one Chinese state (Wuhan), the capital of the province of Hubei. It started in the province as a type of pneumonic disease. The cases were reported to World Health Organization's office in the country and it was discovered to be a new sprain of SARS-CoV 2. The virus was given the name Covid-19 by the W.H.O on February 11th, 2020 (UNDP,2020).

The first confirmed COVID-19 case was on the 27th of February, 2020, at the Infectious Disease Centre, Lagos, Nigeria. He was an Italian national who arrived at Murtala Muhammed International Airport in Lagos about

10:00 p.m. on February 24, 2020, on a Turkish plane from Milan, Italy. The next day, he reported to his company's headquarters in Ogun State and appeared at the company's staff clinic. The on-call doctor was convinced that the virus was present. This prompted him to refer the Italian citizen to the IDH, where the COVID-19 status was confirmed (Nigeria Centre for Disease Control, 2020).

Since the outbreak of COVID-19 in 2019, many authors and researchers have gone to the field in search of solutions from various disciplines using different techniques. A good number of statistical methods have also been employed to track the outbreak to understand its epidemiological behavior and offer useful recommendations to government and policymakers. Chu (2021) investigated the spread of the pandemic in Italy and Spain using the susceptible-infections-recovery model and the log-linear regression model. His results showed the log-linear's ability to predict better fit to the daily incidence cases in both countries. The applications of various time series models and statistical quality tools and control charts have also been found to be useful in monitoring and forecasting the spread of the virus in Nigeria. A study by Edike, Braimah, and Agbedeyi (2020) revealed that as of 18th August 2020, the daily confirmed cases were on a decline. However, they predicted a stationary number of daily confirmed cases using the Pear-Reed logistic model. Ahmad, Aminu, Usman, and Suleiman (2021) estimated the case fatality rate of COVID-19 in Nigeria using both the linear and quadratic polynomial regression models. His work, however, did not point out which of the models had a better fit for the data. A comparative study of models and estimators on the Nigeria covid-19 cases shows the preference of the simple linear regression model, with cases indexed on time, over various other models in their quadratic, cubic and quartic forms (Kayode, Adewale, Rauf, Olusegun, Charles, and Opeyemi, 2020).

This paper investigates the relationship between the cumulative number of covid-19 related deaths and the cumulative number of lab-confirmed cases using the simple linear, linear-log, log-linear, and log-log regression models. The fitness of the models was also examined and the better performing model was determined using the coefficient of determination.

2.0 Materials and Methods

The data used in this study was cumulative number of covid-19 laboratory confirmed cases and the cumulative number of deaths linked to covid-19 in Nigeria from 30th, March 2020 to 10th November 2021. The information was obtained from the Our World in Data (OWID) database (https://ourworldindata.org/coronavirus). The data covers useful corona virus statistics including total cumulative cases, daily new cases, total corona virus-related deaths, and daily corona virus-related deaths. The database comprises COVID-19 statistics for different countries from which that of Nigeria was extracted.

The data collected was analyzed using regression analysis. Scatter plots were used to visualize the nature of the relationship existing between the cumulative covid-19 related deaths and the cumulative number of infected persons. Simple linear, linear-log, log-linear, and log-log regression models were fitted to the data to know which of them provides a better fit, therefore providing better prediction power for the data.

2.1 Model 1 - Simple Linear Regression Model

The linear trend model is as given below:

$$Y_i = \beta_0 + \beta_1 x_i + e \qquad (1)$$

where

Y_i = cumulative number of Covid-19 related death (i = 1, 2, …)

x_i = cumulative Covid-19 laboratory-confirmed cases

β_0 = value of the trend when the cumulative number of lab-confirmed casesx_i= 0

β_1 = the change in Y_i per unit change in the lab-confirmed cases

e = the random error term

2.1.1 Estimation of model parameters

The Ordinary Least Squares (OLS) method is used to estimate the model coefficients.

From equation (1), we obtain the estimate of the model error (the residual) as

$$e = Y_i - \beta_0 - \beta_1 x_i \qquad (2)$$

The sum of squared errors (SSE) is therefore obtained as

$$SSE = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 \qquad (3)$$

Differentiating (3) with respect to the parameters β_0 and β_1, we have

$$\frac{\partial SSE}{\partial \beta_0} = \sum_{i=1}^{n} e_i^2 = -2\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 \qquad (4)$$

$$\frac{\partial SSE}{\partial \beta_1} = \sum_{i=1}^{n} e_i^2 = -2\sum_{i=1}^{n} x_i (y_i - \beta_0 - \beta_1 x_i)^2 \qquad (5)$$

Equating (4) and (5) to zero and simplifying yields the following normal equations

$$\sum_{i=1}^{n} y_i - n\beta_0 - \beta_1 \sum_{i=1}^{n} x_i = 0 \qquad (6)$$

$$\sum_{i=1}^{n} x_i y_i - \beta_0 \sum_{i=1}^{n} x_i - \beta_1 \sum_{i=1}^{n} {x_i}^2 = 0 \qquad (7)$$

Solving (6) for β_0 gives

$$\hat{\beta}_0 = \frac{\sum_{i=1}^{n} y_i}{n} - \frac{\beta_1 \sum_{i=1}^{n} x_i}{n} = \bar{y} - \beta_1 \bar{x} \qquad (8)$$

substituting (8) in (7) gives

$$\hat{\beta}_1 = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2} \qquad (9)$$

where (8) and (9) are the least square estimates of the parameters β_0 and β_1 respectively

2.2 Model 2 – The Linear-log regression model

The linear-log regression model involves the logarithmic transformation of the independent variable (Kenneth, 2011). The log-log regression model is given as:

$$Y_i = \beta_0 + \beta_1 log x_i + e \qquad (10)$$

where

Y_i = cumulative number of Covid-19 related death (i = 1, 2, …).

x_i = cumulative Covid-19 laboratory-confirmed cases.

β_0 = trend value when cumulative number of lab-confirmed cases$x$ = 0.

β_1 = change in Y_i for each unit change in $log x_i$.

e is the random error term.

The ordinary least square estimation method shown under model 1 is also used to estimate the parameters β_0 and $\beta_1$in model 2, and the estimates are as shown in (8) and (9) respectively.

2.3 Model 3 – The Log-linear regression model

The linear-log regression model involves the logarithmic transformation of the dependent variable (Kenneth, 2011). The log-linear regression model is given as:

$$logY_i = \beta_0 + \beta_1 x_i + e \qquad (11)$$

where

Y_i = cumulative number of Covid-19 related death (i = 1, 2, …)

x_i = cumulative number of laboratory confirmed Covid-19 cases

β_0 = value of the trend when cumulative number of lab-confirmed cases$x$ = 0

β_1 = the change in $logY_i$ for each unit change in x_i

e = the random error term

The ordinary least square estimation method shown under model 1 is also used to estimate parameters β_0 and $\beta_1$in model 2, and the estimates are as shown in (8) and (9) respectively.

2.4 Model 4 - Log-log regression model

The log-log regression model is a modification of the simple linear regression model using logarithmic transformation of both variables (Kenneth, 2011). The log-log regression model is given as:

$$logY_i = \beta_0 + \beta_1 logx_i + e \qquad (12)$$

where

Y_i = cumulative number of Covid-19 related death (i = 1, 2, …)

x_i = cumulative number of laboratory-confirmed Covid-19 cases

β_0 = value of the trend when cumulative number of lab-confirmed cases$x$ = 0

β_1 = change in Y_i per unit change in the lab-confirmed cases

e = random error term

Letting $logY_i = y_i$ and $logx_i = x_i$ reduces (10) to the equivalence of (1). Hence the ordinary least square estimation method shown under model 1 is also used to estimate the parameters β_0 and β_1 in model 2, and the estimates are as shown in (8) and (9) respectively.

2.5 Model Performance Comparison

The performances of the fitted models were evaluated and compared using R^2 (coefficient of determination) which shows the variation in proportion, the cumulative death explained by the cumulative total confirmed cases. The higher this proportion, the better the model. The coefficient of determination, usually expressed as a percentage proportion is given in equation (13) below.

$$R^2 = 1 - \frac{SSE}{SST} \qquad (13)$$

where SSE = the sum of squares error, and
SST = sum of squares total

The analyses were carried out using R statistical software and codes were given in the appendix.

2.6 Interpretation of Model Coefficients

Due to the transformation of variables in the models considered, it may be difficult and ambiguous for a layman to understand properly and interpret the actual relationship being examined and presented. In this section, we present the interpretation of the models with respect to a unit change or changes in some standardize values of the independent variable.

2.6.1. The Simple Linear Model

This is the simplest form of the models examined in this paper. Here there are no transformations of variables, hence posing no serious problem interpreting the relationship presented by the model coefficient.

The simple linear model is given as $Y_i = \beta_0 + \beta_1 x_i + e$. Here, a unit change in the independent variable x_i will give a β_1 unit change in the dependent variable Y_i.

2.6.2. The Linear-log Model

In this model, only the independent variable x_i is logarithmically transformed. The Linear-log model is given as $Y_i = \beta_0 + \beta_1 log x_i + e$. In order to interpret the coefficient of this model, we note that a change of one unit in $log x_i$ gives a β_1 unit change in the independent variable Y_i. Hence, we deduce that

$$log x_i + 1 = log x_i + log e$$
$$= log(e x_i)$$

where $e \cong 2.72$

Hence, considering an increase scenario, a unit increase in $log x_i$ translates to multiplying x_i by 2.72. According to Kenneth (2011), it makes more sense to express this change as percentage units instead of log-units. Hence multiplying x_i by 2.72 implies an increase of $(2.72 - 1)100\% = 172\%$.

We can therefore say that a 172% in the independent variable x_i gives a β_1 unit increase in the independent variable Y_i

2.6.3. The Log-linear Model

In this log-linear model, only the dependent variable Y_i is logarithmically transformed. The Log-linear model is given as $log Y_i = \beta_0 + \beta_1 x_i + e$. In order to interpret the coefficient of this model, we note that a change of one unit in x_i gives a β_1 unit change in the independent variable $log Y_i$. Hence, we deduce that:

$$Y_i = exp(\beta_0 + \beta_1 x_i + e)$$
$$= exp(\beta_0) exp(\beta_1 x_i)\, exp(e)$$

Hence a unit increase in x_i would give

$$Y_i = exp(\beta_0) exp[\beta_1(x_i + 1)]\, exp(e)$$
$$= exp(\beta_0) exp(\beta_1 x_i + \beta_1)\, exp(e)$$
$$= exp(\beta_0) exp(\beta_1 x_i)\, exp(\beta_1) exp(e)$$

This shows that using the log-linear model, a unit increase in x_i multiplies the expected value of Y_i by $exp(\beta_1)$.

2.6.4. The Log-log Model

In this model, both variables are logarithmically transformed. The Log-log model is given as $log Y_i = \beta_0 + \beta_1 log x_i + e$. Kenneth (2011) pointed that using the log-log regression model, the interpretation will be a combination of the linear-log and the log-linear regression models. This implies that multiplying x_i by e will multiply the dependent variable Y_i by $exp(\beta_1)$.

3.0 Analysis and Results

The findings of this study are presented in this section. First, a scatter plot of the data collected is shown in figure 1while the scatter plot for its logarithmic transformation is shown in Figure 2.

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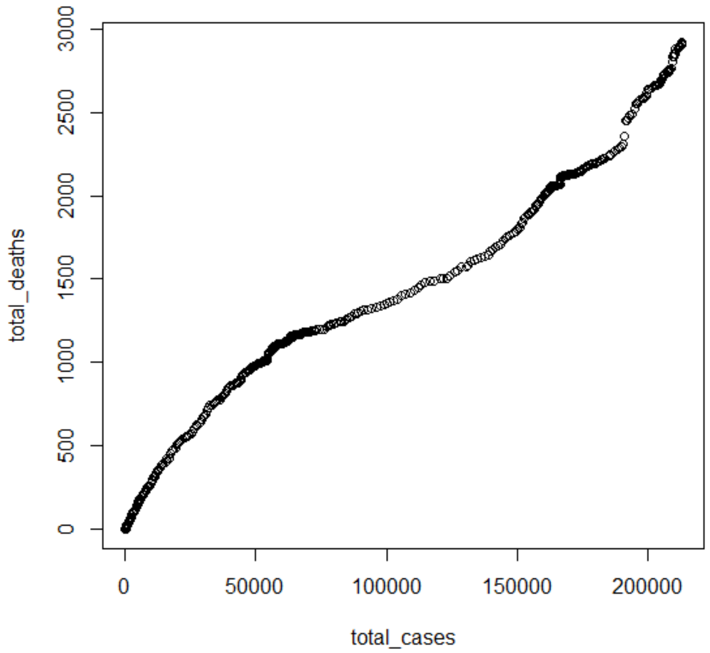

Figure 1: Scatter plot of the cumulative number of death and cumulative number cases confirmed

The scatter plot in figure 1 above shows a linear relationship between the cumulative number of covid-19 related deaths and the cumulative number of total confirmed cases.

3.1 Regression Analysis of cumulative no. of death on cumulative no. of confirmed cases using the simple linear regression model

Model 1 as shown in section 2 was fit to the data as shown below. From the model, using equations (8) and (9), the OLS estimate of β_0 is 254.00548, while that of β_1 is 0.01145. Hence,

$$cumm_{death} = 254.00548 + 0.01145 cumm_{cases} \quad (14)$$

the fitted regression model.

The regression line in Equation (14) shows that, on average, one additional confirmed covid-19 case would translate to additional 0.01145 covid-19 related death in Nigeria.

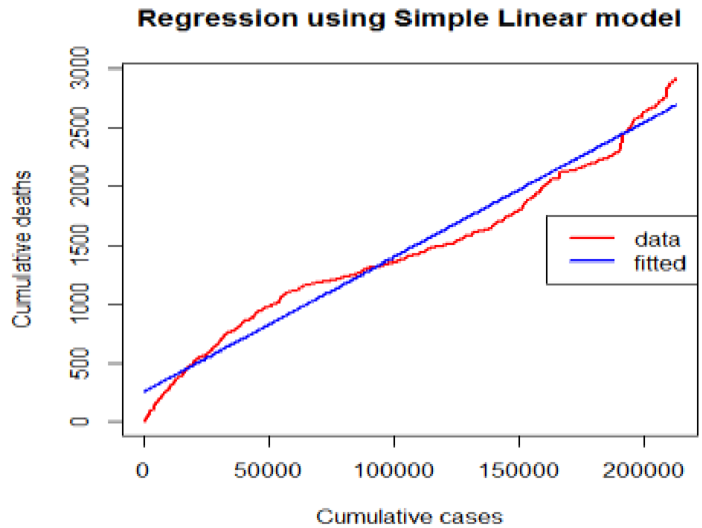

Figure 2: Simple Linear Regression Plot showing the comparison of data and fitted model

The significance of the fitted model was tested using Analysis of Variance in Table 1

Table 1: Analysis of Variance (ANOVA) Table for Model 1

Source	DF	SS	MS	F	P
Regression	1	384296291	384296291	21093	2.2e-16
Error	595	10840575	18219		
Total	596	395136866			

Table 1 above shows the Analysis of Variance (ANOVA) of the fitted regression model. The table reveals an F-ratio of 21093 with a p-value of 2.2e-16, which is an indicator that the fitted model is significant at a 5% level of significance.

Coefficient of Determination

Using equation (5), we obtained the coefficient of determination as:

$$R^2 = 1 - \frac{10840575}{395136866} = 0.9725$$

The fitted model gave a coefficient of determination (R^2) of 0.9725. This shows that, in this model, about 97 percent of the variation in cumulative number of covid-19 related death is explained by the cumulative number of total covid-19 confirmed cases.

3.2 Regression Analysis of cumulative no. of death on cumulative no. of confirmed cases using Linear-log regression model

Model 2 as shown in section 2 was fit into the data as shown below. From the model, using equations (8) and (9), the OLS estimate of β_0 is -3042, while that of β_1 is 412. Hence, the fitted linear-log regression model is:

$$cumm_{death} = -3042 + 412 \ \ logcumm_{cases} \tag{15}$$

The regression line in Equation (13) shows that, on average, one additional unit of log confirmed covid-19 case would translate to additional 412 covid-19 related death in Nigeria. It is important to state here that making predictions using this model would require the conversion to the normal numeric scale by taking the anti-logarithm of the final results.

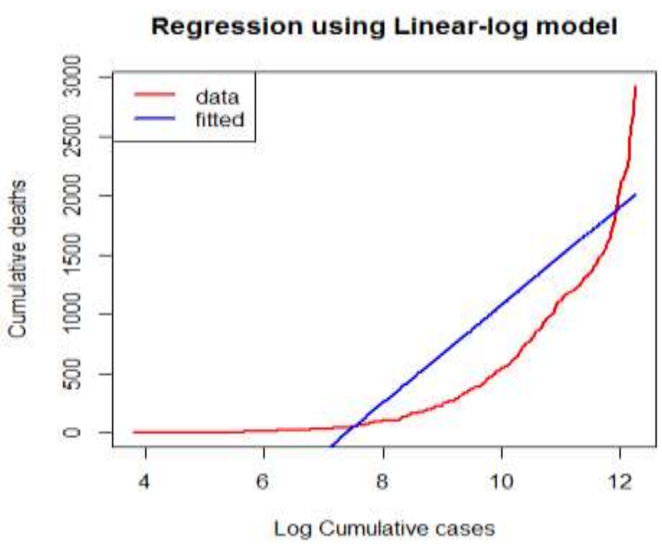

Figure 3: Linear-log regression Plot showing the comparison of data and fitted model

The significance of the fitted model is tested using Analysis of Variance as Table 2 below depicts.

Table 2: Analysis of Variance (ANOVA) Table for Model 2

Source	DF	SS	MS	F	P
Regression	1	279208354	279208354	1433	2.2e-16
Error	595	115928512	194838		
Total	596	395136866			

Table 2 above shows the Analysis of Variance (ANOVA) of the fitted regression model. The table reveals an F-ratio of 1433 with a p-value of 2.2e-16, which is also an indicor that the fitted model is significant at a 5% level of significance.

Coefficient of Determination

Using equation (5), we obtained the coefficient of determination as:

$$R^2 = 1 - \frac{115928512}{395136866} = 0.7066$$

The fitted model gave a coefficient of determination (R^2) of 0.7066. This shows that, in this model, about 71 percent of the variation in cumulative number of covid-19 related death is explained by the cumulative number of total covid-19 confirmed cases.

3.3 Regression Analysis of cumulative no. of death on cumulative no. of confirmed cases using log-linear regression model

Model 3 as shown in section 2 was fit into the data as shown below. From the model, using equations (8) and (9), the OLS estimate of β_0 is 5.257, while that of β_1 is1.497e-05. Hence, the fitted linear regression model is:

$$log\ cumm_{death} = 5.257 + 1.497e - 05\ \ cumm_{cases} \quad (16)$$

The regression line in Equation (16) shows that, on average, one additional confirmed covid-19 case would translate to an additional 0.0000149 log of covid-19 related death in Nigeria. It is important to state here that making predictions using this model would require the conversion to the normal numeric scale by taking the anti-logarithm of the dependent variable.

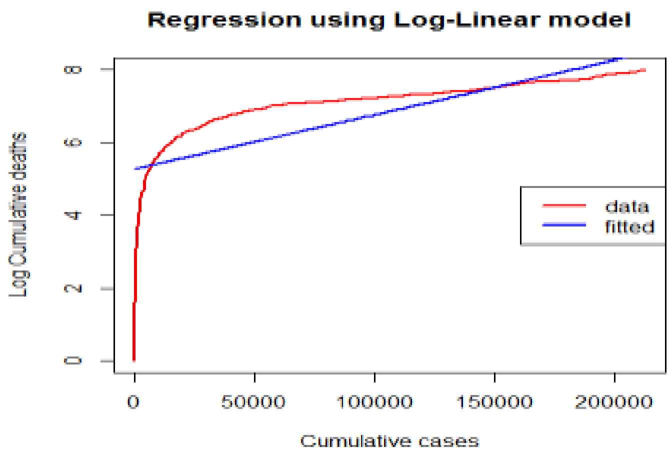

Figure 4: Log-linear regression Plot showing the comparison of data and fitted model

The significance of the fitted model is tested using Analysis of Variance as Table 3 below depicts.

Table 3: Analysis of Variance (ANOVA) Table for Model 3

Source	DF	SS	MS	F	P
Regression	1	656.33	656.33	633.11	2.2e-16
Error	595	616.82	1.04		
Total	596	1273.15			

Table 3 above shows the Analysis of Variance (ANOVA) of the fitted regression model. The table reveals an F-ratio of 633.11 with a p-value of 2.2e-16, indicating the fitted model is significant at a 5% level of significance.

Coefficient of Determination

Using equation (5), we obtained the coefficient of determination as:

$$R^2 = 1 - \frac{616.82}{1273.15} = 0.5155$$

The fitted model gave a coefficient of determination (R^2) of 0.5155. This shows that, in this model, about 52 percent of the variation in collective number of covid-19 related death is explained by the cumulative number of total covid-19 confirmed cases.

3.4 Regression Analysis of cumulative no. of death on cumulative no. of confirmed cases using log-log regression model

Model 4 as shown in section 2 was fit into the data as shown below. From the model, using equations (8) and (9), the OLS estimate of β_0 is -2.6995, while that of β_1 is 0.8703. Hence, the fitted linear regression model is:

$$logcumm_{death} = -2.6995 + 0.8703\ \ logcumm_{cases} \quad (17)$$

The regression line in Equation (13) shows that, on average, one additional confirmed covid-19 case on the log scale would translate to an additional 0.8703 log of covid-19 related death in Nigeria. It is important to state here that making predictions using this model would require the conversion to the normal numeric scale by taking the anti-logarithm of the final results.

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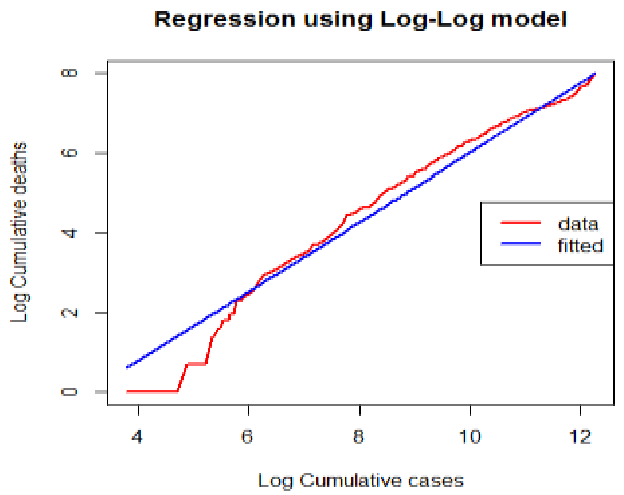

Figure 5: Fitted Regression Line of log_cumulative no. of death on log_cumulative no. of confirmed cases

The significance of the model 4was tested using Analysis of Variance as Table 4 below depicts.

Table 4: Analysis of Variance (ANOVA) Table for Model 4

Source	DF	SS	MS	F	P
Regression	1	1246.04	1246.04	27352	2.2e-16
Error	595	27.11	0.05		
Total	596	1273.15			

Table 4 above shows the Analysis of Variance (ANOVA) of the fitted regression model 4. The table reveals an F-ratio of 27352 with a p-value of 2.2e-16. This also an evidence that model 4 is also significant at a 5% level of significance.

Coefficient of Determination

Using equation (5), we obtained the coefficient of determination as:

$$R^2 = 1 - \frac{27.11}{1273.15} = 0.9787$$

The fitted model gave a coefficient of determination (R^2) of 0.9787. This shows that, in this model, about 98 percent of the variation in cumulative number of covid-19 related death is explained by the cumulative number of total covid-19 confirmed cases.

3.5 Performance comparison of models

Models	Simple Linear	Linear-Log	Log-Linear	Log-Log
R-squared	0.9725	0.7066	0.5155	0.9787

4.0 Discussion of Results

The regression model in its various variable transformation forms has been used to model the fatality of the covid-19 pandemic in Nigeria. The simple linear, the linear-log, the log-linear, and the log-log regression models were fitted to the collected data. Though all four models showed significance at a 5% level of significance, further analysis, using the coefficient of determination showed that all the models did not perform equally. The simple linear regression gave a coefficient of determination (R^2) of 0.9725, the linear-log model gave an R-squared of 0.7066, the log-linear, 0.5155 while the log-log regression model gave an R-squared value of 0.9787. With about 98 percent of the variation in the cumulative number of covid-19 related deaths explained by the cumulative number of lab-confirmed cases, the log-log regression model is said to have better performance compared with the other three models.

5.0 Conclusion

This paper is on the modeling the fatality of the covid-19 pandemic in Nigeria intending to find if there is any relationship between total cumulative number of confirmed cases and the total cumulative number of death. It also examined the fitness and performance of four regression models on the data collected. The simple linear regression model, linear-log, the log-linear, and the log-log regression model were examined. The result showed that the log-log regression model with an R-square of 0.978 provides a better fit to the data, hence a better prediction power than the other three competing models.

Data

Hannah Ritchie, Edouard Mathiew, Lucas Rodes-Guirao, Cameron Apel, Charlie Giattino, Esteban Ortiz-Ospina, Joe Hasell, Bobbie Macdonald, Diana Beltekian and Max Roser (2020) – "Coronavirus Pandemic (COVID-19)". Published online at OurWorldInData.org. Retrieved from: 'https://ourworldindata.org/coronavirus' [Online Resource]

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APPENDIX: R codes

```
#MODELS
model1<-lm(covid$cum_deaths~covid$cum_cases)
model2<-lm(covid$cum_deaths~covid$log_cum_cases)
model3<-lm(covid$log_cum_deaths~covid$cum_cases)
model4<-
lm(covid$log_cum_deaths~covid$log_cum_cases)

#SUMMARIES
summary(model1)
summary(model2)
summary(model3)
summary(model4)

#ANOVA TABLES
anova(model1)
anova(model2)
```

anova(model3)
anova(model4)