Academic Journal of Statistics and Mathematics (AJSM)
Vol.5, No.7; July - 2019;
ISSN (5730 – 7151);
p –ISSN 4052 – 392X
Impact factor: 3.79

SENSITIVITY ANALYSIS OF LASSA FEVER MODEL WITH PERSONAL PROTECTION AND TREATMENT CONTROLS

Obasi, Chinedu and Mbah, G.C.E

Department of Mathematics, University of Nigeria, Nsukka

Corresponding author: Chinedu Victor Obasi,

Abstract: *Lassa fever is a deadly disease transmitted through ingestion of food that is contaminated with infected rodent's saliva, urine or excreta, infected person and inhalation of the aerosol. In this paper, we investigated the sensitivity analysis of the transmission dynamics of Lassa fever model with personal protection and treatment controls earlier constructed. The existence of the steady state of the model was determined, after which the effective reproduction number of the model was computed using the next generation matrix approach. Sensitivity analysis was then carried out to determine which parameters that should be targeted by intervention strategies. The result revealed that increase in personal protection, treatment rate and rate of recovery with temporary immunity leads to a reduction in the spread of Lassa fever.*

Keywords: Lassa fever, Modeling, Equilibrium points, Stability, Sensitivity Analysis

Introduction

Lassa fever is an acute viral illness cause by Lassa virus. It belongs to the member of Arenavirus family. The disease was first described in the 1950s and the virus was identified in 1969, when two missionary nurses died from it in the town of Lassa in Borno state, Nigeria. Lassa fever is endemic in West African countries such as Guinea, Liberia, Sierra Leone and Nigeria [6]. Studies show that about 500, 000 cases of Lassa fever occur per year in West Africa with approximately 5000 death [12]. The animal host of the Lassa virus is the rodents called Mastomys Natalensis [3]. Currently, there is an outbreak of Lassa fever in Nigeria and 63 lives were claimed out of 212 suspected cases reported and about 17 states were affected [2].

Lassa fever can be transmitted through ingestion of food that is contaminated with infected rodent's saliva, urine or excreta, inhalation of the aerosol as occurs during the sweeping of an area where the droppings are present, contaminated needles or exposure to an infected aerosol in health care setting and person-to-person transmission occurs through body fluid exchange. The symptoms of Lassa fever take up to three weeks to manifest. They start with a lower fever, tiredness, body ache, sore throat and headache, nausea, vomiting and even diarrhea, high fever, swelling of the face, bleeding from the eyes or nose, breathing difficulties, severe pain in the back, chest and abdomen, shock, etc. There is no US approved vaccine for Lassa fever but it can be treated using Ribavirin which is effective during early stage of infectiousness [11]. Lassa fever can be prevented by using: rodent-proof containers for food storage should be used, rodents control measures such as traps and insecticides are to be used in and around human homes, avoid eating rodent (rats), avoid attracting rodents to house by cleanliness and healthy waste disposal practices, isolation of patients till recovery is well advanced, use of gown, gloves mask and cap, careful segregation of biologically hazardous waste and sterilizing all equipment used for the patients.

This Lassa fever epidemic, with its increased frequency indicates that our current knowledge in its dynamics and public health guidelines to control the disease is not adequate. The study is carried out by the use of mathematical model which is formulated based on the knowledge of Lassa fever

outbreak. Hence, the present study will construct a mathematical model on the spread of the Lassa fever disease dynamics with current features.

Mathematical models play a vital role in analyzing the spread and control of different diseases. In fact, epidemiological modeling helps us to understand the dynamics of these diseases so that we can trace the most important parameter(s) responsible for the disease spread [5]. It is important to mention that mathematical modeling is very crucial in epidemiology since in most cases we cannot do experiments. It is possible to mathematically model the progress of most infectious diseases to discover the likely outcome of an epidemic or to help manage them by different control programmes. Mathematical model is a powerful tool to investigate the interior of the Lassa fever virus transmission. In this paper a compartmental mathematical model is constructed to investigate the sensitivity analysis of Lassa fever transmission dynamic model with controls.

Sensitivity analysis in mathematical biology tells us how important each parameter is to disease transmission and is used to assess how sensitive a model is to variations in the value of the parameters of the model and to changes in the structure of the model [4]. It refers to technique used to determine how independent variable values will impact a particular dependent variable under a given set of assumptions. It is used to determine parameters that have high impact on the basic reproduction number so that it is directly targeted by intervention strategies. It is necessary to determine how sensitive the threshold quantity basic reproductive number with respect to its parameters. This analysis reveals how crucial each of the parameter is to the disease transmission. Sensitivity analysis has been used for various parameterization tasks of models of biological systems, such as finding essential parameters for research prioritization [7], identifying insignificant parameters or model reduction or parameters clustering [10].

Classically, sensitivity of the model is determined by the partial derivatives of the outcome with respect to its parameters. Sensitivity analysis methods based on such quantities are called local as the derivative is taken at a fixed point in the state space of model parameters – at the parameter face value. Moreover, these methods belong to the class of one-factor-at-a-time (OAT) methods, because the net effect of a parameter on the property of the outcome is taken while assuming that all other factors are fixed [10].

In recent times, sensitivity analysis of Lassa fever models has received attention of researchers in mathematical epidemiology and has become a subject of intense study. A good number of mathematical models has been developed to assess the sensitivity of each factor driving the disease in order to determine the major factors to be targeted by appropriate interventions in order to eradicate Lassa fever. The present work is aimed at conducting sensitivity analysis on a new Lassa fever model incorporating aerosol transmission mode as well as personal protection and treatment control strategies for effective management of Lassa fever spread.

Lassa fever Mathematical Model [5]

The model for the transmission dynamics of Lassa fever incorporating protection and treatment control strategies is as given in [5]) and presented in (1). The associated variables and parameters of the model are described in Tables 1 and 2.

Academic Journal of Statistics and Mathematics (AJSM)
Vol.5, No.7; July - 2019;
ISSN (5730 – 7151);
p –ISSN 4052 – 392X
Impact factor: 3.79

$$\frac{dS_h}{dt} = \Lambda_h + \phi(1-v)I_h + \psi R_h - (1-u_1)\beta_1\sigma I_r S_h - (1-u_1)\beta_2\varepsilon I_h S_h - \eta(1-u_1)(1-e^{-rt})S_h - \mu_h S_h$$
$$\frac{dE_h}{dt} = (1-u_1)\beta_1\sigma I_r S_h + (1-u_1)\beta_2\varepsilon I_h S_h + \eta(1-u_1)(1-e^{-rt})S_h - (\kappa+\mu_h)E_h$$
$$\frac{dI_h}{dt} = \kappa E_h - \phi(1-v)I_h - (\phi+u_2)vI_h - (\delta+\mu_h)I_h$$
$$\frac{dR_h}{dt} = (\phi+u_2)vI_h - (\psi+\mu_h)R_h \quad (1)$$
$$\frac{dS_r}{dt} = \Lambda_r - \beta_3\vartheta\rho(1-u_1)I_h S_r - u_3(1-\rho)S_r - \mu_r S_r$$
$$\frac{dE_r}{dt} = \beta_3\vartheta\rho(1-u_1)I_h S_r - u_3(1-\rho)E_r - (\alpha+\mu_r)E_r$$
$$\frac{dI_r}{dt} = \alpha E_r - u_3(1-\rho)I_r - \mu_r I_r$$

Table 1: Description of the state variables of the model

Variable	Description
S_h	Number of Susceptible humans
E_h	Number of Exposed humans
I_h	Number of Infectious humans
R_h	Number of Recovered humans
S_r	Number of Susceptible rodents
E_r	Number of Exposed rodents
I_r	Number of Infectious rodents

Table 2: Description of the parameters of the basic Lassa fever model

Parameters	Description
Λ_h	Recruitment level of humans
Λ_r	Recruitment level of rodents
δ	Per capita Lassa-induced death rate
ψ	Recovered human loss of immunity
ϕ	Spontaneous individual recovery
β_1	Transmission rate per contact by an infectious rodent
β_2	Transmission rate per contact by an infective through sexual activity
β_3	Transmission rate per contact by an infected human
η	Relative infectiousness of individuals with aerosol
μ_h	Natural mortality rate for humans
μ_r	Natural mortality rate for rodents
κ	Progression rate of human from exposed to infected
α	Progression rate of rodents from exposed to infected
σ	Contact rate of rodent per human per unit time

Academic Journal of Statistics and Mathematics (AJSM)
Vol.5, No.7; July - 2019;
ISSN (5730 – 7151);
p –ISSN 4052 – 392X
Impact factor: 3.79

ϑ	Relative human-to-rodent transmissibility of infected humans
ε	Relative human-to-human transmissibility of infected humans
r	Rate of exposure to aerosol
v	Recovery with temporary immunity
u_1	Personal protection rate
u_2	Treatment rate
u_3	Rate of spray of rodenticide
ρ	Fraction of rodent population reduced

The system (1) has a disease-free equilibrium (DFE) given by

$$\xi_0 = \left(S_h^*, E_h^*, I_h^*, R_h^*, S_r^*, E_r^*, I_r^*\right) = \left(\frac{\Lambda_h}{\eta\left(1-u_1\right)\left(1-e^{-rt}\right)+\mu_h}, 0, 0, 0, \frac{\Lambda_r}{u_3\left(1-\rho\right)+\mu_r}, 0, 0\right)$$

The local stability of the DFE (ξ_0) can be established using the next generation method approach in [5] and the effective reproduction number, denoted by R_{eff}, can be computed and is given as

$$R_{eff} = \frac{\left(1-u_1\right)\beta_2\varepsilon\kappa\Lambda_h}{\left(\kappa+\mu_h\right)\left(\phi+\nu u_2+\mu_h+\delta\right)\left(\eta\left(1-u_1\right)\left(1-e^{-rt}\right)+\mu_h\right)}$$

The result below follows Theorem 2 in [5].

Lemma 1: The DFE (ξ_0) of the model (1) is locally asymptomatically stable (LAS) if $R_{eff} < 1$, and unstable if $R_{eff} > 1$.

Sensitivity Analysis

Sensitivity indices allow us to measure the relative change in a variable when a parameter changes. If the model is simple, it may be possible to differentiate the outcome with respect to each parameter in turn [7]. The derivatives are the rate of changes of predictions with respect to the parameters. This work adopts the normalized forward sensitivity index to conduct the sensitivity index of a variable with respect to a parameter is the ratio of the change in the parameter. When the variable is a differentiable function of the parameter, the sensitivity index may be alternatively defined using partial derivative [10]. For instance, the normalized forward sensitivity index on R_{eff}, which depends differentially on a parameter φ, is defined by

$$\Pi_{\varphi}^{R_{eff}} = \frac{\partial R_{eff}}{\partial \varphi} \times \frac{\varphi}{R_{eff}} \qquad (2)$$

The sensitivity index of parameters with respect to R_{eff} is given as:

Academic Journal of Statistics and Mathematics (AJSM)
Vol.5, No.7; July - 2019;
ISSN (5730 – 7151);
p –ISSN 4052 – 392X
Impact factor: 3.79

$$\Pi_{\Lambda_h}^{R_{eff}} = \frac{(1-u_1)\beta_2\varepsilon\kappa}{(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)} \times \frac{\Lambda_h(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)}{(1-u_1)\beta_2\varepsilon\kappa\Lambda_h} = 1 > 0$$

$$\Pi_{\beta_2}^{R_{eff}} = \frac{(1-u_1)\varepsilon\kappa\Lambda_h}{(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)} \times \frac{\beta_2(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)}{(1-u_1)\beta_2\varepsilon\kappa\Lambda_h} = 1 > 0$$

$$\Pi_{\varepsilon}^{R_{eff}} = \frac{(1-u_1)\kappa\Lambda_h}{(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)} \times \frac{\varepsilon(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)}{(1-u_1)\beta_2\varepsilon\kappa\Lambda_h} = 1 > 0$$

$$\Pi_{\kappa}^{R_{eff}} = \frac{(1-u_1)\beta_2\varepsilon\Lambda_h\mu_h}{(\kappa+\mu_h)^2\left[(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)\right]} \times \frac{\kappa(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)}{(1-u_1)\beta_2\varepsilon\kappa\Lambda_h}$$

$$= \frac{\mu_h}{(\kappa+\mu_h)} = \frac{0.02}{0.01+0.02} = 0.667 > 0$$

$$\Pi_{u_1}^{R_{eff}} = \frac{-\beta_2\varepsilon\kappa\Lambda_h\mu_h}{(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left[\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)\right]^2} \times \frac{u_1(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)}{(1-u_1)\beta_2\varepsilon\kappa\Lambda_h}$$

$$= \frac{-\mu_h u_1}{(1-u_1)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)} = \frac{-0.02(0.6)}{(0.4)(0.03(0.4)(0.0198)+0.02)} = -1.482 < 0$$

$$\Pi_{u_2}^{R_{eff}} = \frac{-(1-u_1)\beta_2\varepsilon\Lambda_h\nu}{(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)^2\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)} \times \frac{u_2(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)}{(1-u_1)\beta_2\varepsilon\kappa\Lambda_h}$$

$$= \frac{-\nu u_2}{(\phi+\nu u_2+\mu_h+\delta)} = \frac{-0.08(0.6)}{0.001+0.08(0.6)+0.02+0.2} = -1.784 < 0$$

$$\Pi_{\nu}^{R_{eff}} = \frac{-(1-u_1)\beta_2\varepsilon\Lambda_h u_2}{(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)^2\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)} \times \frac{\nu(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)}{(1-u_1)\beta_2\varepsilon\kappa\Lambda_h}$$

$$= \frac{-\nu u_2}{(\phi+\nu u_2+\mu_h+\delta)} = \frac{-0.08(0.6)}{0.001+0.08(0.6)+0.02+0.2} = -1.784 < 0$$

$$\Pi_{\phi}^{R_{eff}} = \frac{-(1-u_1)\beta_2\varepsilon\kappa\Lambda_h}{(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)^2\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)} \times \frac{\phi(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)}{(1-u_1)\beta_2\varepsilon\kappa\Lambda_h}$$

$$= \frac{-\phi}{(\phi+\nu u_2+\mu_h+\delta)} = \frac{-0.001}{0.001+0.08(0.6)+0.02+0.2} = -0.004 < 0$$

Academic Journal of Statistics and Mathematics (AJSM)
Vol.5, No.7; July - 2019;
ISSN (5730 – 7151);
p –ISSN 4052 – 392X
Impact factor: 3.79

$$\Pi_{\delta}^{R_{eff}} = \frac{-(1-u_1)\beta_2\varepsilon\kappa\Lambda_h}{(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)^2\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)} \times \frac{\delta(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)}{(1-u_1)\beta_2\varepsilon\kappa\Lambda_h}$$

$$= \frac{-\delta}{(\phi+\nu u_2+\mu_h+\delta)} = \frac{-0.2}{0.001+0.08(0.6)+0.02+0.2} = -0.743 < 0$$

$$\Pi_{\eta}^{R_{eff}} = \frac{-(1-u_1)^2(1-e^{-rt})\beta_2\varepsilon\kappa\Lambda_h}{(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)^2} \times \frac{\eta(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)}{(1-u_1)\beta_2\varepsilon\kappa\Lambda_h}$$

$$= \frac{-\eta(1-u_1)(1-e^{-rt})}{\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)} = \frac{-0.03(0.4)(0.0198)}{0.03(0.4)(0.0198)+0.02} = -0.0012 < 0$$

$$\Pi_{\mu_h}^{R_{eff}} = \frac{-(1-u_1)\beta_2\varepsilon\kappa\Lambda_h\left[3\mu_h+2\mu_h\left(\left(2\kappa(\phi+\nu u_2+\delta)+\eta(1-u_1)(1-e^{-rt})\right)+\kappa(\phi+\nu u_2+\delta)+2\kappa(\phi+\nu u_2+\delta)\left(\eta(1-u_1)(1-e^{-rt})\right)\right)\right]}{\left[(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)\right]^2}$$

$$\times \frac{\mu_h(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)}{(1-u_1)\beta_2\varepsilon\kappa\Lambda_h}$$

$$= \frac{-\mu_h\left[3\mu_h+2\mu_h\left(\left(2\kappa(\phi+\nu u_2+\delta)+\eta(1-u_1)(1-e^{-rt})\right)+\kappa(\phi+\nu u_2+\delta)+2\kappa(\phi+\nu u_2+\delta)\left(\eta(1-u_1)(1-e^{-rt})\right)\right)\right]}{(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)} = -15.603 < 0$$

$$\Pi_{r}^{R_{eff}} = \frac{(1-u_1)^2\beta_2\eta\varepsilon\kappa\Lambda_h e^{-rt}}{t(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)^2} \times \frac{r(\kappa+\mu_h)(\phi+\nu u_2+\mu_h+\delta)\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)}{(1-u_1)\beta_2\varepsilon\kappa\Lambda_h}$$

$$= \frac{\eta(1-u_1)re^{-rt}}{t\left(\eta(1-u_1)(1-e^{-rt})+\mu_h\right)} = \frac{0.0002352}{0.0202376} = 0.0116 > 0$$

Table 3: Sensitivity indices of the effective reproduction number to model parameters

Parameter	Estimated values [2]	Sensitivity index
ν	0.08	–1.784
u_2	0.6	–1.784
u_1	0.6	–1.482
Λ_h	20	1
β_2	0.002	1
ε	0.5	1
κ	0.01	–0.667
r	0.02	0.0116
η	0.03	–0.0012

Academic Journal of Statistics and Mathematics (AJSM)
Vol.5, No.7; July - 2019;
ISSN (5730 – 7151);
p –ISSN 4052 – 392X
Impact factor: 3.79

It is observed in Table 2 that these parameters have either positive or negative effects on the effective reproduction number. Positive index indicates that the value of R_{eff} will increase with increase in the parameter and the one with negative index will decrease the value of R_{eff} with increase in the parameter values.

Discussion of Results

From Table 2, the most significant parameters affecting the effective reproduction number are personal protection rate, treatment rate and recovery with temporary immunity. These parameters should be targeted by control intervention strategies. Increasing these parameters will decrease the effective reproduction number for the control model. Similarly if the values are decreased, the effective reproduction number will be increased and as such the spread of Lassa fever increases. An increase in the positively indexed parameters like rate of exposure to aerosol, transmission rate per contact by an infective through sexual activity and relative human-to-human transmissibility rate of infected humans increase the value of the effective reproduction number and as such the disease invades the population. The results in Table 2 have far reaching implications to Lassa fever transmission and control. Firstly, it shows that personal protection plays an indispensable role in curtailing the spread of Lassa fever outbreak. This collaborates the earlier works of [8] and [9] who concluded that the best strategies against the disease are maintaining good community hygiene to control the mastomys and taking care of the infective under specific isolation precaution. This result shows that with individuals' total adherence to effective use of personal protection, little treatment efforts will then be required by the community in the control of the spread of the disease. Secondly, the lowest sensitivity index, which is the relative infectiousness of individuals with aerosol does not imply that this parameter is not relevant to Lassa fever control. It implies that Lassa fever prevention is achievable provided that relative infectiousness of individuals with aerosol is low enough to cause outbreak. This could be done by keeping the environment very ventilated. Ventilation represents a primary infectious disease control strategy through dilution of room air around a source and removal of infectious agents [1].

Conclusion

In this paper, we presented a Lassa fever control model using a deterministic system of differential equations and established that the model is locally asymptotically stable when the effective reproduction number is less than unity. The effective reproduction number is stated and the sensitivity analysis is performed. The sensitivity indices of the effective reproduction number with respect to the key parameters are evaluated to determine the relative importance of the parameters to the disease transmission and control and the result is presented in tabular form. In communities where resources are scarce, we suggest that the combination of treatment and personal protection should be adopted, having observed from the comparison of basic and effective reproduction numbers.

References

[1] Centre for Disease Control (CDC) (2005). Guidelines for preventing the transmission of mycobacterium tuberculosis in health-care settings. *Morbidity and Mortality Weekly Report* (MMWR), 54 (No. RR-17):1–140. Atlanta: Centers for Disease Control and Prevention.

[2] Central Intelligence Agency (2015). World fact book for the year 2014. Retrieved on 20 February 2016 from *http:/www.cia/library/publication/the-world fact book/geos/ni.htm*

[3] Fisher-Hoch, S.P., Tomori, O., Nasidi, A., Perez-Oronoz, G.I., Fakile, Y., & Hutwagner, L. (1995). Review of cases of nosocomial Lassa fever in Nigeria: The high price of poor medical practice. *Biomedical Journal*, 311, 857–869.

[4] Numfor, E. S (2010). Mathematical modelling, simulation and time series analysis of seasonal epidemics. *M.Sc Thesis, East Tenessee State University*.

[5] Obasi, C (2018). Developing and analysing mathematical models of transmission and controls of Lassa fever dynamics. *A PhD seminar presented to the Department of Mathematics, University of Nigeria, Nsukka*.

[6] Ogbu,O. E., Ajuluchukwu, C. J.,& Uneke, C.J. (2007). Lassa fever in West Africa sub-region: An overview. *Journal of Vector Borne Diseases* 44, 1-11.

[7] Okosun, K & Smith, R (2017). Optimal control analysis of malaria-schistosomiasis co-infection dynamics. *Math. Bio-sci. Engineer*. 14 (2), 337-405.

[8] Okuonghae, D. & Okuonghae, R. (2006). A Mathematical model for Lassa fever. *Journal of the Nigerian Association of Mathematical Physics*, 10,457-464.

[9] Onuorah, M .O., Ojo, M .S., Usman, D.J & Ademu, A (2016). Basic reproductive number for the spread and control of Lassa fever. *International Journal of Mathematics Trends and Technology* (IJMTT), 30 (1), 1-7.

[10] Peter, O.J et al (2018). Sensitivity analysis of the parameters of a cholera model. *J. Appl. Sci. Environ. Manage.* 22 (4), 477-481.

[11] Tara, K. H. (2004). Virology notes in Lassa fever. Retrieved on March 10, 2012 from *www.taraharper.com/v lass.html*

[12] World Health Organization (2004). Centre for disease control. Imported Lassa fever. *Morbidity Mortal Weekly Reports*, 53(38), 894-897.