



## ESTIMATION OF GOMPERTZ GROWTH MODEL ON MORTALITY RATE OF CHILDREN UNDER FIVE YEARS OF AGE IN NIGERIA THAT SUFFERED FROM HIV/AIDS AND DIARRHOEA

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**Abstract:** This work analyses the mortality rate of children under five years in Nigeria for a period of 15 years. The data was obtained from United Nations International Children’s Education Fund (UNICEF) on HIV and diarrhoea mortality rate. A transformed Gompertz model was used for the analysis. The result show that as the mortality growth rate for both HIV and diarrhoea is increased the growth range of the mortality also increased. The best fitted model for predicting mortality for 0-12 months old children was seen to be  $y = 29126e^{0.3954e^{0.75x}}$  and  $y = 85814e^{-0.6598e^{0.25x}}$  for HIV and diarrhoea respectively, while the best fitted model for predicting mortality rate for children between the ages 1-5 years was found to be  $y = 29421.8e^{0.3960e^{0.75x}}$  and  $y = 87811.7e^{-0.6576e^{0.25x}}$  for HIV and diarrhoea respectively. The model predicted an increase in mortality due to diarrhoea and a steady mortality due to HIV.

**Keywords:** Gompertz Growth Model, Logarithm Transformation Method, Least Square Estimation Method, Mortality Rate (HIV/AIDS and Diarrhoea).

### 1. INTRODUCTION

The Gompertz function is a sigmoid function that was first applied by Mr. Benjamin Gompertz in 1825 (Panik, 2014), in describing growth data. He fitted it to the relationship between increasing death rate and age, what he referred to as “the average exhaustions of a man’s power to avoid death”, or the “portion of his remaining power to oppose destruction” (Panik, 2014). Gompertz function is a type of mathematical model for a time series, where growth is slowest at the beginning and ending of a period. The issue that usually is confusing is the fact that the Gompertz function can be presented in different mathematical forms. But for this work we applied the form given below;

$$y = ae^{-be^{-cx}} \quad (1)$$

The equation (1) is known as the double-exponential form. The parameters are defined as;

- e represents Euler’s number
- x represent time

a represent upper asymptote when time approaches  $+\infty$

b represents growth rate (or shape parameter)

c represents growth range (or intrinsic growth range)

Y is the observation number of deaths

Other forms of the function are given below;

$$y = ae^{-bc^x} \quad (2)$$

$$y = y_0 e^{\frac{r}{\alpha}(1-e^{-\alpha x})} \quad (3)$$

Where equation (2) is the single exponential which is usually easier to fit and its parameters are defined as in equation (1). The next expression, equation (3) is usually limited to biomathematics. Where  $y_0$  is the initial ( $t=0$ ) number of individuals and in our case initial number of infant mortality.

Health priority since the United Nations (UN) Millennium Declaration has focused strongly on Children under five (5) years of age (Zhang *et al.*, 2015).



The Declaration on Protection and Preservation of Life and Wellbeing of Children in the World was passed in the 1990 and the goal was to reduce the mortality rate of Children under five years of age to two-thirds by 2015 (UNICEF, 2017). Since then, Child mortality has been at the centre of health dialogue.

Health professionals, Government and Policy makers globally, have displayed an exclusive interest in the fight to reduce the predominance of Child fatalities (Morakinyo and Fagbamigbe, 2017). This evolution has resulted in the development of sturdy processes to reducing mortality among Children under five years of age between 1990 and 2015, and between 2015 and 2030 as projected by the United Nations (UN) Millennium Development Goals (MDGs) (Fox, 2012) and Sustainable Development Goals (SDGs) respectively (United Nation, 2015). Neonatal mortality has reportedly been a global public health challenge. For instance, in 2015, 2.7 million deaths occurred in the first 28 days of an infant life, recording a significant reduction of 19 deaths per 1000 live births from previous 36 deaths per 1000 live births in 1990 (UNICEF, 2015). Though Lawn et al., (2014), noted that there has been a decrease in neonatal deaths in some African Sub-Sahara countries like Ghana, Uganda and Nigeria with Nigeria Neonatal Mortality Rate (NMR) reducing by 20.4% from 49 deaths per 1000 live births in 1990 to 39 in 2011, 37 in 2013 to 34 deaths per 1000 live births in 2015 placing Nigeria second to India as the highest number of neonatal deaths. Furthermore, analysis of recent trends shows that Nigeria has made progress in reducing infant and under-five deaths, nevertheless, the mortality rate of Children under-five in Nigeria remains high irrespective of the developments in reducing Child mortality rates. According to Nigeria Demographic Health Survey (2013), about one in every eight children in Nigeria dies before their fifth birthday. Reports shows that mortality rate of Children under five years of age usually results from several risks factors or preventable or treatable infectious diseases such as Respiratory Infections (ARLs), diarrhoea, malaria, pneumonia measles, HIV/AIDS which account for more than 70% of the estimated one million under-five deaths in Nigeria

(Wani, 2017). Additionally, other factors such as such as regional variation, malnutrition, poor breastfeeding practices, lack of maternal education, lack of use of health services by mother and/or child etc. also causes the death of under-five children (Morakinyo and Fagbamigbe, 2017). Since the recorded progress in achieving the recommended United Nation's Millennium Development Goals has been insufficient in meeting the target globally and in Nigeria, this research investigates the mortality rate of children under the age of five in Nigeria using *Gompertz growth model* and tries to evaluate progress towards achieving the UN Millennium Development Goals.

**1.1 HIV/AIDS:** The human immune deficiency virus (HIV) is a lentivirus (a subgroup of retrovirus) that causes HIV infection and over time acquired immune deficiency syndrome (AIDS). AIDS is a condition in humans in which progressive failure of the immune system allows life-threatening opportunistic infections and cancers to thrive (Young *et al.*, 2007). Without treatment, average survival time after infection with HIV is estimated to be 9 to 11 years, depending on the HIV subtype (Young *et al.*, 2007). Infection with HIV occurs by the transfer of blood, semen, vaginal fluid, pre-ejaculate, or breast milk (Song *et al.*, 2017).

**1.2 DIARRHOEA:** Diarrhoea, is the condition of having at least three loose or liquid bowel movements each day (Saengcharoen and Lerkiatbundit, 2010). It often lasts for a few days and can result in dehydration due to fluid loss. Signs of dehydration often begin with loss of the normal stretchiness of the skin and irritable behaviour (Saengcharoen and Lerkiatbundit, 2010). This can progress to decreased urination, loss of skin colour, a fast heart rate, and a decrease in responsiveness as it becomes more severe (Swinger, 2002). The most common cause is an infection of the intestines due to a virus, bacteria, or parasite; a condition known as gastroenteritis (Bhutta *et al.*, 2008). These infections are often acquired from food or water that has been contaminated by stool, or directly from another person who is infected (Bhutta *et al.*, 2008).



### 3. METHODOLOGY

The data used in this research is data obtained from United Nations International Children’s Education Fund (UNICEF) database on the causes of death in Nigeria for children under five through HIV and diarrhoea from 2000 to 2015. The research is limited to Gompertzian model with respect HIV/AIDS and diarrhoea death for children under five years in Nigeria.

The statistical software’s used are Microsoft excel and statistical index.

#### 3.1 GOMPERTZ GROWTH MODEL

The data collected were analysed using Gompertz Growth Model, the Gompertz model applied is given in equation (1) as;

$$y = ae^{-be^{-cx}} \quad c \text{ and } b > 0 \tag{4}$$

By applying logarithm transformation, we have

$$\ln y = \ln a - be^{-cx} \tag{5}$$

Transforming it to linear regression model, we let  $\ln(a)=\beta_0$  and  $-b= \beta_1$  be the parameters to be estimated at  $c = 0.25$  and  $0.50$  and  $0.75$ .

Hence equation (3.2) becomes;

$$\ln y = \beta_0 + \beta_1 e^{-cx} \tag{6}$$

Therefore, by least square estimation method of a simple linear regression, the parameter are estimates by

Equation (3.4) and (3.5) at  $\beta= [0.25, 0.50 \text{ and } 0.75]$ . That is

$$\beta_1 = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} \tag{7}$$

$$\beta_1 = \bar{y} - \beta_1 \bar{x} \tag{8}$$

where,  $y = \ln y$  and  $x = e^{-cx}$

The purpose of applying this approach was to capture wide range of raw textual data in other to provide an in-depth understanding of the mortality rate of children under the age of five in Nigeria using *Gompertz growth model* and evaluate progress towards achieving the UN Millennium Development Goals.

### 4. RESULTS

In this section the gompertzian model of mortality rate of children under five years the death is generated and the relationship that exists between the dependent and independent variables are analysed. The significance of the gompertz growth model coefficients is also determined. A program developed using Micro-excel is designed to calculate the estimates of the gompertz model. Using Equation (3.3) at  $c = 0.25$  and  $0.50$  and  $0.75$ , we estimated the gompertz growth model parameters for ages 0-12 months and 1-5 years.

The gompertz growth model parameters,  $R^2$  and AIC for 0-12 months and 1-5 years, are summarised in Table 4.1 at  $c = 0.25$  and  $0.50$  and  $0.75$ .

**TABLE 4.1: Gompertz growth model parameters and  $R^2$  results for ages 0-12 months**

Variable	Growth Rate (c)	Asymptote (a )	Growth range ( b)	$R^2$	AIC
HIV	0.75	29126.0	0.3954	<b>0.414909*</b>	<b>-40.7001*</b>
HIV	0.50	29255.1	0.2756	0.36762	-40.0782
HIV	0.25	29409.7	0.1472	0.19227	-38.2787
DIARRHOEA	0.75	94181.6	0.8844	0.438118	-28.5968
DIARRHOEA	0.50	92036.5	0.7539	0.580784	-30.9446
DIARRHOEA	0.25	85814.2	0.6598	<b>0.81685*</b>	<b>-37.5693*</b>



The results from table 4.1 above shows that as the mortality growth rate for HIV is increased the growth range of the mortality also increased. We also observed from the values  $R^2$  and AIC that the model also was improving as the mortality growth range increased, giving the model  $y = 29126e^{-0.3954e^{0.75x}}$  as the best fitted model. Also for mortality through diarrhoea, we observed that as the mortality growth rate was increased the mortality growth range also increased, but from the values  $R^2$  and AIC the model was seen to be improving as the mortality growth range decreased, giving the model:  $y = 85814.2e^{-0.6598e^{0.25x}}$  as the best fit.

**TABLE 4.2: Prediction based on best fitted model for 0-12 months**

Year	Number of deaths from HIV	Number of deaths from DIARRHOEA
2025	29119	81291
2030	29125	84494
2035	29125	85434
2045	29126	85783

The table (4.2) above shows that the model predicts an increase in mortality of children under 5 years from diarrhoea and a steady mortality rate from HIV for children between 0-12 months.

**TABLE 4.3: Gompertz growth model parameters  $R^2$  and AIC results for ages 1-5 years**

Variable	Growth Rate (c)	Asymptote (a)	Growth Range (b)	$R^2$	AIC
HIV	0.75	29421.8	0.3960	<b>0.415224*</b>	<b>-40.7069*</b>
HIV	0.50	29551.2	0.2757	0.367935	-40.0848
HIV	0.25	29707.65	0.1473	0.192513	-38.1252
DIARRHOEA	0.75	96345.9	0.8813	0.43884	-28.6799
DIARRHOEA	0.50	94157.1	0.7506	0.582018	-31.0357
DIARRHOEA	0.25	87811.7	0.6576	<b>0.818489*</b>	<b>-36.9453*</b>

The result from table 4.3 above shows that as the growth rate of mortality through HIV is increased the growth range of the mortality also increased. We also observed from the values  $R^2$  and AIC that the model also was improving as the mortality growth range increased, giving the model  $y = 29421.8e^{0.3960e^{0.75x}}$  as the best model. But for mortality through diarrhoea, we observed also that as the mortality growth rate is increased the mortality growth range also increased, also from the values  $R^2$  and AIC the model was seen to be improving as the mortality growth range decreased, giving the model:  $y = 87811.7e^{-0.6576e^{0.25x}}$  as the best fit with 81.84% variation in the model accounted for.

TABLE 4.4 Prediction based on best fitted model for 1-5 years

Year	Number of deaths from HIV	Number of deaths from DIARRHOEA
2025	29415	83204
2030	29422	86465
2035	29422	87424
2045	29422	87779

The table (4.4) above shows that the model predicts an increase in mortality of children under 5 years from diarrhoea and a steady mortality rate from HIV for children between 1-5 years.

### 5. Conclusion

This study was able to employ the gompertz model which is a non-linear model to analyse the mortality rate of children under-five years from 0-12 months and 1-5 years for a period of 15 years. The estimates of the parameters with an initial growth rate value of  $c = 0.25$ ,



0.50 and 0.75 were applied to determine the behaviour of the disease growth rate. The transformed gompertz model was estimated for both age ranges and best fitted models were obtained. The models predict an increase in mortality rate due to diarrhoea and a steady mortality rate due to HIV. We also observed that more children between the ages of 1-5 years die more from diarrhoea than those between 0-12 months. This research work has shown that there is little or no progress in achieving the UN Millennium Development goals of reducing child mortality in Nigeria. More needs to be done especially for children between the ages of 1-5 years, as their predicted deaths is seen to be more than those between the ages of 0-12 months.

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