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EFFECT OF HEAT AND MASS TRANSFER ON UNSTEADY MHD OSCILLATORY FLOW OF FLUID IN A VERTICAL POROUS CHANNEL

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Abstract: In this paper, we investigated the effect of heat and mass transfer on unsteady MHD oscillatory flow of fluid in a vertical porous channel. The flow is laminar and incompressible. The temperatures prescribed at the channel walls are non – uniform. A uniform magnetic field is applied transverse to the channel. Closed form solution method is used to solve the dimensionless equations that govern the flow and the solutions for velocity, temperature and concentration distribution are obtained. The influence of flow parameters as they affect the velocity profile, temperature distribution and species concentration are analysed and shown graphically in detail through graphs. Out of many results, it is concluded that the suction/injection parameter accelerates the velocity and elevates the temperature distribution and species concentration.

Keywords: MHD, oscillatory flow, heat and mass transfer, closed form, porous channel **Nomenclature**

t'	Time
u'	Velocity of the fluid
v_0	Uniform velocity
ρ	Density of the fluid
P'	Pressure of the fluid
β	Volumetric expansion due to heat transfer
β^*	Volumetric expansion due to concentration
C_p	Specific heat at constant pressure
α	Thermal radiation term
k	Thermal conductivity
T'	Fluid temperature
T_0	Referenced temperature
C'	Concentration of the fluid
K_r'	Chemical reaction
D	Mass diffusivity.

1. Introduction

The study of heat and mass transfer on unsteady (MHD) oscillatory flow of fluid in a vertical porous channel effects are important in different areas such as MHD generators, arterial blood flow, petrochemical engineering etc.

There are several studies on the transfer of heat and mass in oscillatory flow problems such as Joseph *et al.* [1], Joseph *et al.* [2], *Falade et al.* [3], Dulal and Sukanta [4], Makinde and Mhone [5], Mehmood and Ali [6], Chauchau and Kumar [7], Idowu *et al*. [8] and Daniel *et al*. [9]. Palani and Abbas [10] investigated the combined effects of magneto-hydrodynamics and the effect of radiation on the free convection flow that passes through an isothermal vertical plate impulsively initiated using the Rosseland approach. Hussain *et al*. [11] presented an analytical study of the second degree oscillatory fluid flow in the presence of a transverse magnetic field. Idowu *et al.* [12] studied the effect of heat and mass transfer on the unstable MHD oscillatory flow of Jeffrey's fluid in a horizontal channel with chemical reaction. The temperature prescribed in the plates is uniform and asymmetric. A perturbation method is used to solve the momentum, energy and concentration equations. Skin frictions, Nusselt numbers and Sherwood numbers are evaluated using the perturbation technique. The effects of several dimensionless parameters in the velocity and temperature profiles are considered and discussed in detail through graphs and tables. Umavathi *et al.* [13] investigated the unstable flow of viscous fluid through a horizontal composite channel whose average width is filled with porous medium. Adesanya and Makinde [14] investigated the effect of radiation heat transfer on the pulsatile couple's stress fluid flow with a time-dependent limit condition on the heated plate. They affirmed that the non-slip condition is not realistic in some flows that involve nano-channels, micro-channels and flows on plates coated with hydrophobic substances. Krishna [15] studied the heat transport of copper and alumina nanofluids past a stretching porous surface. Also, Krishna *et al.* [16] considered heat and mass transfer on unsteady, magnetohydrodynamic, oscillatory flow of second grade fluid through a
porous medium between two vertical plates, under the influence of fluctuating heat source/sink and chemical reaction.

2. Problem formulation

We consider an electrically viscous incompressible fluid through a vertical porous channel. The flow is laminar and unstable with the condition of no sliding on the heated plate and sliding on the cold plate. A magnetic field B_0 of uniform force is applied transversely to the channel. It is assumed that the fluid has a small electrical conductivity and that the electromagnetic force produced is very small. The flow is subjected to suction in the cold wall and injection into the hot wall. The flow of the fluid is along the vertical direction under a chemical reaction with species concentration C' as shown in figure 1. The channel width is twice $y' = a$.

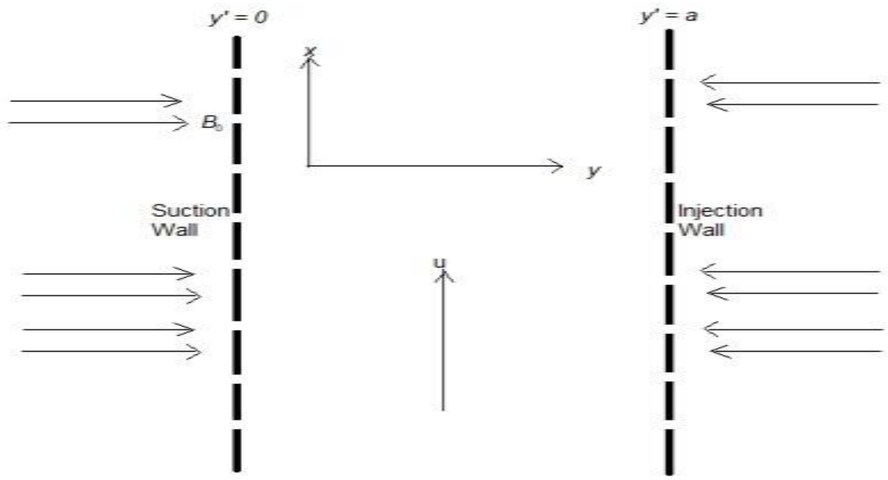

Figure 1: Physical diagram of the problem (Joseph *et al.* [1])

The governing equations of the flow field subject to Bousinesq approximation are

$$\frac{\partial u'}{\partial t'} - v_0\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{dP}{dX} + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\nu}{K}u' - \frac{\sigma_e B_0^2}{\rho}u' + g\beta(T' - T_0') + g\beta^*(C' - C_0') \tag{1}$$

$$\frac{\partial T'}{\partial t'} - v_0\frac{\partial T'}{\partial y'} = \frac{k_f}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{4\alpha^2}{\rho C_p}(T' - T_0') \tag{2}$$

$$\frac{\partial C'}{\partial t'} - v_0\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

With the boundary conditions

$$u' = \frac{\sqrt{K}}{\alpha_s}\frac{du'}{dy'},\ T' = T_0',\ C' = C_0' \text{ on } y' = 0 \tag{4}$$

$$u' = 0, T' = T_1',\ C' = C_1' \text{ on } y' = a \tag{5}$$

In order to write equations (1) to (5) in dimensionless form we use the following dimensionless parameters and variables

$$x' = \frac{x'}{h}, y' = \frac{y'}{h}, u = \frac{hu'}{\nu}, t = \frac{\nu t'}{h^2}, P = \frac{h^2 P}{\rho\nu^2}, Gr = \frac{g\beta(T_0' - T_0')h^3}{\nu^2}, Gc = \frac{g\beta^*(C_0' - C_0')h^3}{\nu^2}, Pr = \frac{\rho C_p \nu}{k}, \theta = \frac{T' - T_0'}{T_1' - T_0'}, \phi = \frac{C' - C_0'}{C_1' - C_0'}, \delta = \frac{4\alpha^2 h^2}{\rho C_p \nu}, \gamma = \frac{\sqrt{K}}{\alpha_s h}, Ha^2 = \frac{\sigma_e B_0^2 h^2}{\rho\nu}, Da = \frac{K}{h^2}, s = \frac{v_0 h}{\nu}, Sc = \frac{\nu}{D} \tag{6}$$

Equations (1) to (5) become

$$\frac{\partial u}{\partial t} - s\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\partial^2 u}{\partial y^2} - \left(Ha^2 + \frac{1}{Da}\right)u + Gr\theta + Gc\phi \tag{7}$$

$$\frac{\partial\theta}{\partial t} - s\frac{\partial\theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2} + \delta\theta \quad (8)$$

$$\frac{\partial\phi}{\partial t} - s\frac{\partial\phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2} \quad (9)$$

The boundary conditions now become

$$\left.\begin{array}{l} u(0) = \gamma\frac{du(0)}{dy}, \theta(0) = 0, \phi(0) = 0 \\ u(1) = 0, \theta(1) = 1, \phi(1) = 1 \end{array}\right\} \quad (10)$$

3. Solution of the Problem

For purely oscillatory flow, the solutions of the dimensionless equations (7) to (9) are presented as follows

$$-\frac{dP}{dx} = \lambda e^{i\omega t}, u(y,t) = u_0(y)e^{i\omega t}, \theta(y,t) = \theta_0(y)e^{i\omega t}, \phi(y,t) = \phi_0(y)e^{i\omega t} \quad (11)$$

Where λ is any positive constant and ω is the frequency of oscillation.
Substituting equation (11) into equations (7) – (11), we have the following differential equations

$$\frac{\partial^2 u_0}{\partial y^2} + s\frac{\partial u_0}{\partial y} - A_2 u_0(y) = -\lambda - Gr\theta_0(y) - Gc\phi_0(y) \quad (12)$$

$$\frac{\partial^2\theta_0}{\partial y^2} + A_3\frac{\partial\theta_0}{\partial y} + A_4\theta_0(y) = 0 \quad (13)$$

$$\frac{\partial^2\phi_0}{\partial y^2} + s\frac{\partial\phi_0}{\partial y} - A_5\phi_0(y) = 0 \quad (14)$$

The boundary conditions in equation (10) becomes

$$\left.\begin{array}{l} u_0(0) = \gamma\frac{du_0}{dy}, \theta_0(0) = 0, \phi_0(0) = 0 \\ u_0(1) = 0, \theta_0(1) = 0, \phi_0(1) = 0 \end{array}\right\} \quad (15)$$

Solving equations (12) to (14) subject to the boundary conditions (15) and using equation (11), the solutions for the non – periodic terms are as follows;

$$u0(y) = C1em1y + C2em2y + K0 + K1em1y + K2em2y + K3em3y + K4em4y \quad (16)$$

$$\theta_0(y) = C_3e^{m^3y} + C_4e^{m^4y} \quad (17)$$

$$\phi_0(y) = C_5e^{m^1y} + C_6e^{m^2y} \quad (18)$$

Therefore, the analytical solutions for velocity, temperature and concentration distribution are

$$u(y,t) = (C1em5y + C2em6y + K0 + K1em1y + K2em2y + K3em3y + K4em4y)ei\omega t \quad (19)$$

$$\theta(y,t) = (C_3 e^{m^3 y} + C_4 e^{m^4 y}) e^{i\omega t} \tag{20}$$

$$\phi(y,t) = (C_5 e^{m^1 y} + C_6 e^{m^2 y}) e^{i\omega t} \tag{21}$$

Where,

$$A_1 = Ha^2 + \frac{1}{Da}, A_2 = A_1 + i\omega, A_3 = sPr, A_4 = Pr(\delta - i\omega), A_5 = i\omega, C_5 = \frac{1}{e^{m_1} - e^{m_2}}, C_6 = -c_1, c_3 = \frac{1}{e^{m_3} - e^{m_4}}, C_4 = -C_3, c_5 = \frac{A_8 - A_{10}c_6}{A9}, C_1 = \frac{A_{10}e^{m_5} - A_9 e^{m_6}}{A_{10}e^{m_5} - A_9 e^{m_6}}, m_1 = \frac{-s + \sqrt{s^2 + 4A_5}}{2}, m_2 = \frac{-s - \sqrt{s^2 + 4A_5}}{2}, m_3 = \frac{-A_3 + \sqrt{A_3^2 - 4A_4}}{2}, m_4 = \frac{-A_3 - \sqrt{A_3^2 - 4A_4}}{2}, m_5 = \frac{-s + \sqrt{s^2 + 4A_2}}{2}, m_6 = \frac{-s - \sqrt{s^2 + 4A_2}}{2}, K_0 = \frac{\lambda}{A_2}, K_1 = \frac{-GcC_5}{m_1^2 + sm_1 - A_2}, K_2 = \frac{-GcC_6}{m_2^2 + sm_2 - A_2}$$

4. Discussion of Results

The effect of heat and mass transfer on unsteady MHD oscillatory flow fluid in a vertical porous channel has been analysed, the velocity u, the temperature θ and the concentration of species ϕ plot profiles are plotted against y and for different values of the parameter of flow parameters.

Fig. 2. And **Fig. 3** demonstrate the effect of suction s on fluid velocity u and temperature profile θ. It is seen that both the velocity u and temperature θ increase with increase in suction parameter s. This means that the difference between the external and internal pressure enhances the flow of the fluid. This also show the porosity of the channel.

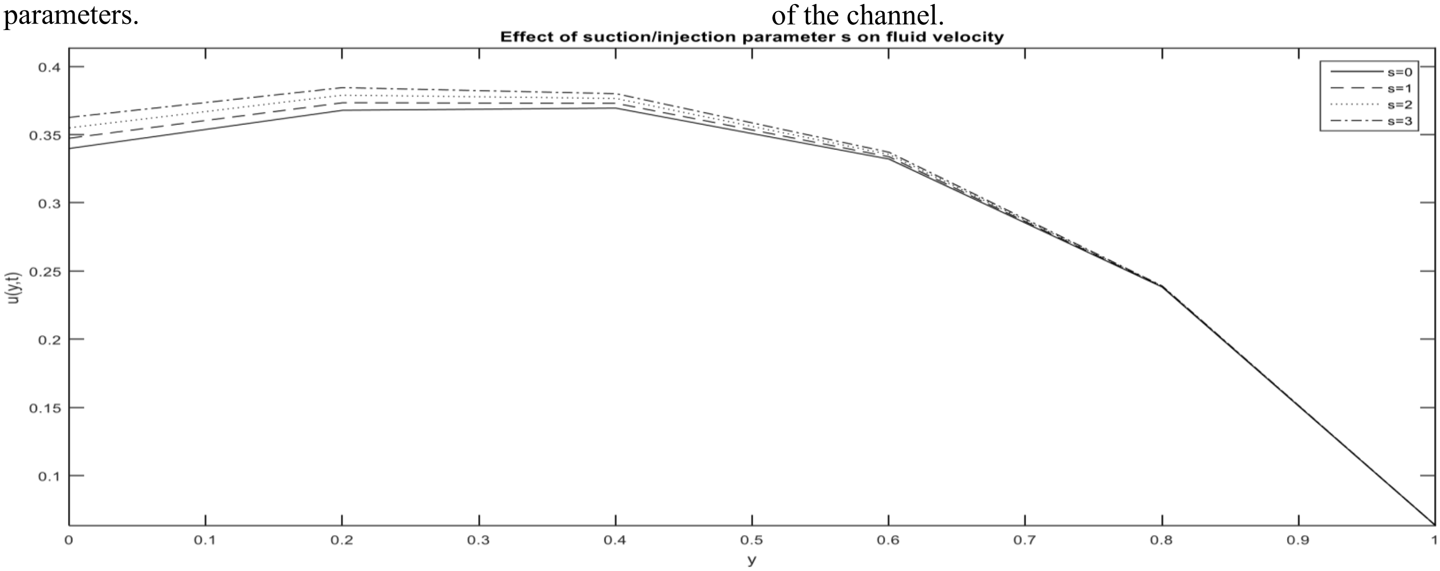

Fig. 2.: Plot of velocity $u(y,t)$ against y showing influence s on u with $Da = 1$, $\delta =$

1, $Ha = 1$, $\lambda = 1$, $\gamma = 0.1$, $Gr = 5$, $Gc = 5$, $\omega = 0.5$, $Pr = 0.71$, $Sc = 1$ and $t = 0.01$

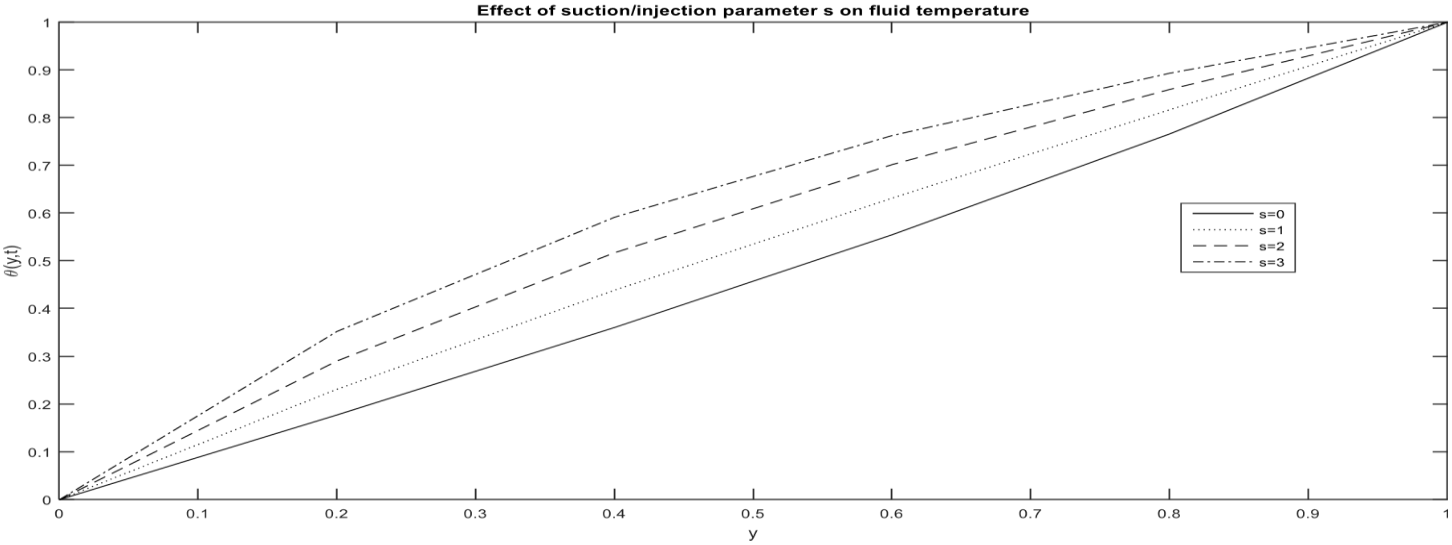

Fig. 3.: Plot of temperature $\theta(y, t)$ against y showing effect s on θ with $\delta = 1$, $Pr = 0.71$, $\omega = 0.5$ and $t = 0.01$

The impact of Hartmann number is illustrated in **Fig. 4**. The velocity profile decreases with the increment of Hartmann number. The role of Hartmann number which is the magnetic parameter is to suppress turbulence. Physically, when magnetic field is applied to any fluid, the apparent viscosity of the fluid increases to the point of becoming viscous elastic solid. It is of great interest that yield stress of the fluid can be controlled very accurately through variation of the magnetic field intensity. The result is that the ability of the fluid to transmit force can be controlled with help of electromagnet which give rise to many possible control – based applications, including MHD power generation, electromagnetic casting of metals, MHD propulsion etc.

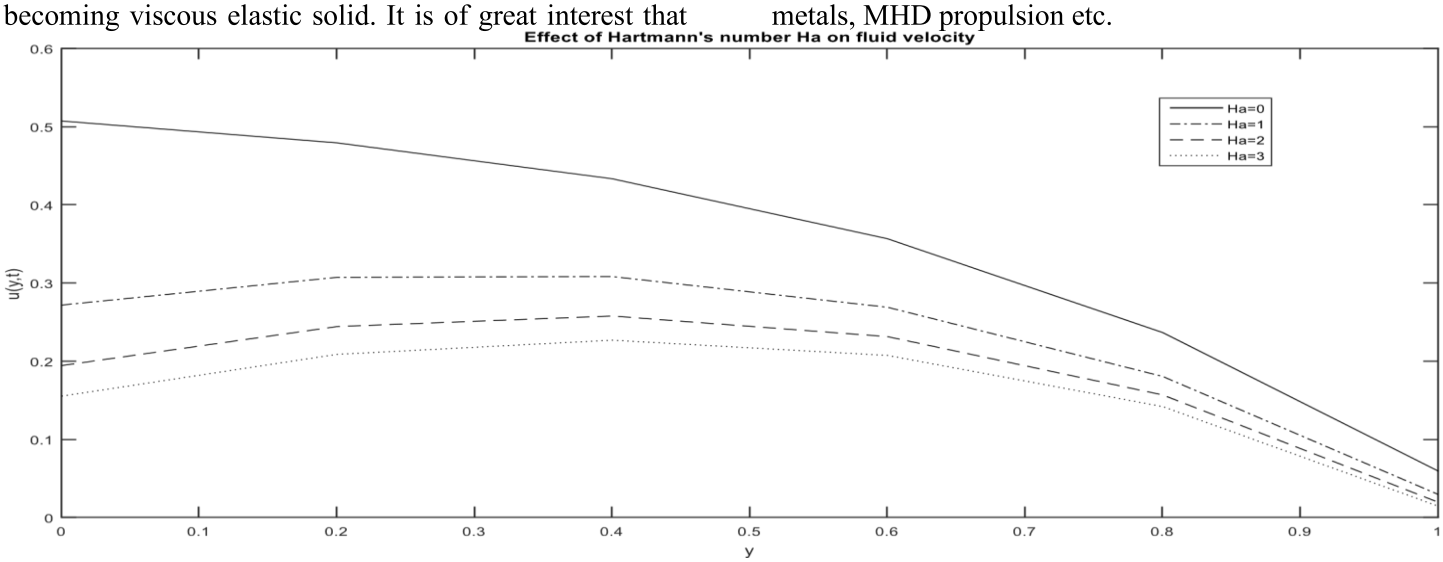

Fig. 4.: Plot of velocity $u(y, t)$ against y showing influence of Hartmann's number Ha on fluid

velocity with $Gc = 1$, $Pr = 0.71$, $\gamma = 1$, $s = 1$, $Gr = 1$, $\delta = 1$, $\omega = 0.5$, $\lambda = 1$, $Da = 1$, $Sc = 1$ and $t = 0.01$

Fig. 5. and **Fig. 6.** demonstrate the effect of Grashof number Gr due to heat transfer and Grashof number Gc due to mass transfer on fluid velocity u respectively. It is evident that the velocity increases as both Gr and Gc increases.

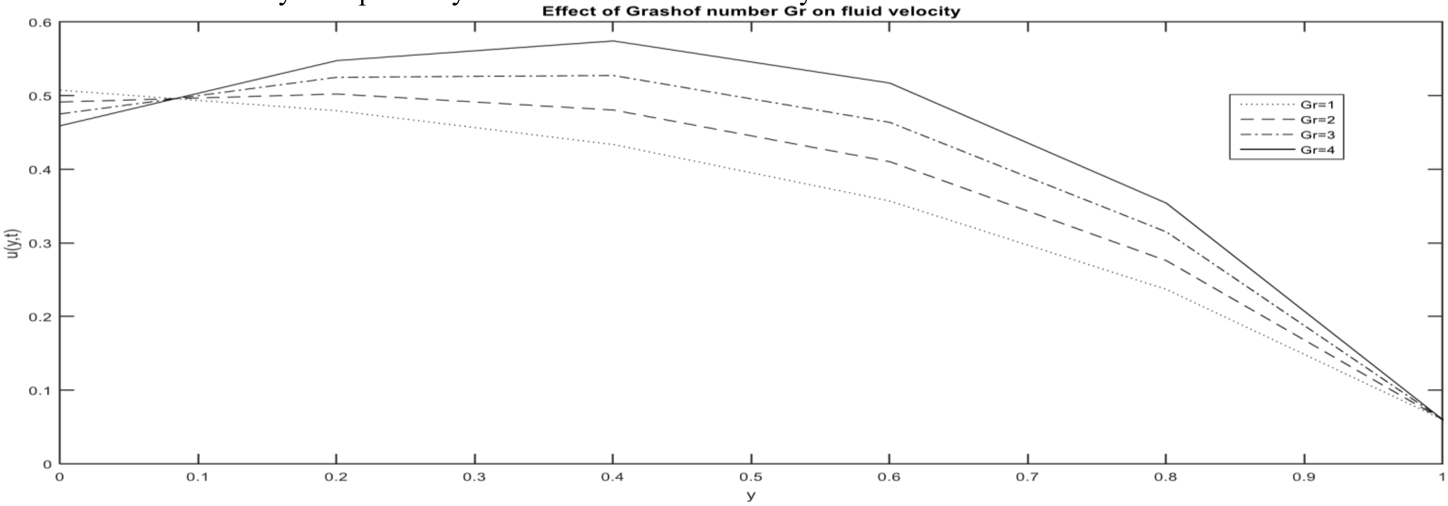

Fig. 5.: Plot of velocity $u(y, t)$ against y showing influence of Gr on u with $\lambda = 1$, $\gamma = 1$, $s = 1$, $Da = 1$, $Ha = 1$, $\delta = 1$, $Gc = 5$, $\omega = 0.5$, $Pr = 0.71$, $Sc = 1$ and $t = 0.01$

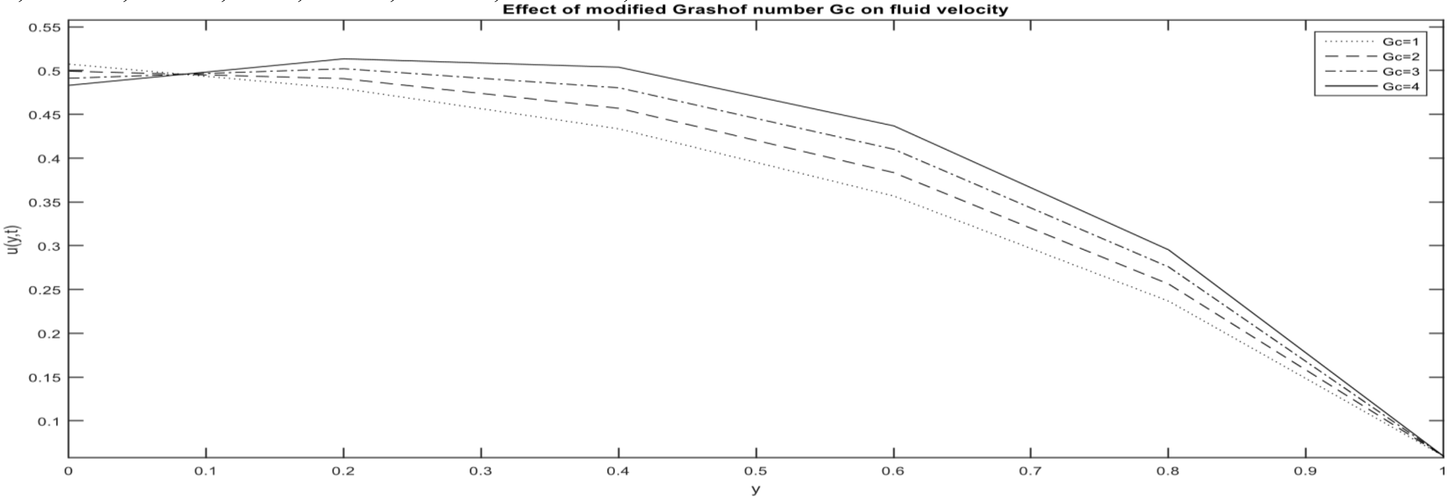

Fig. 6.: Plot of velocity $u(y, t)$ against y showing influence of Gc on u with $Ha = 1$, $Gr = 5$, $\lambda = 1$, $\gamma = 1$, $s = 1$, $Da = 1$, $\delta = 1$, $\omega = 0.5$, $Pr = 0.71$, $Sc = 1$ and $t = 0.01$

The effect of thermal radiation δ on fluid velocity u is demonstrated in **Fig. 7.**. Velocity of the fluid u increases as δ increases.

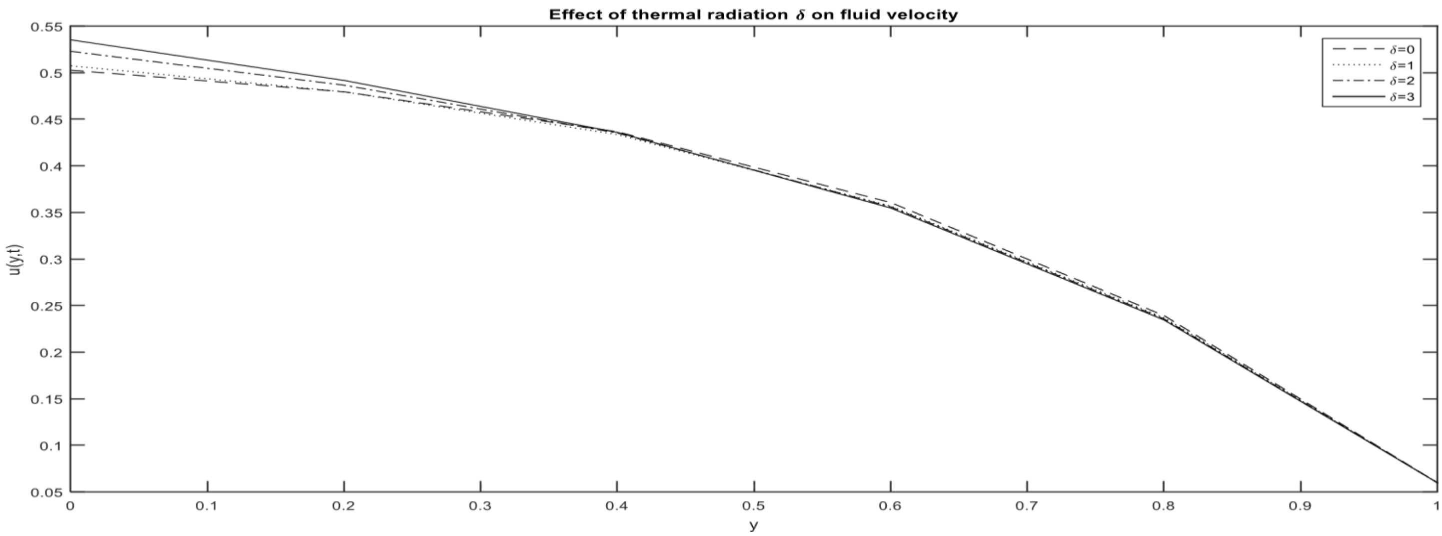

Fig. 7.: Plot of velocity $u(y, t)$ against y showing influence of thermal radiation δ on fluid velocity with , $Gr = 5$, $Gc = 5$, $\lambda = 1$, $\gamma = 1$, $s = 1$, $Ha = 1$, $Da = 1$, $Gc = 1$, $\omega = 0.5$, $Pr = 0.71$, $Sc = 1$ *and* $t = 0.01$

Fig. 8. and **Fig. 9.** show the effects of δ and ω respectively. It is evident that increase in both the flow parameters decrease the fluid temperature.

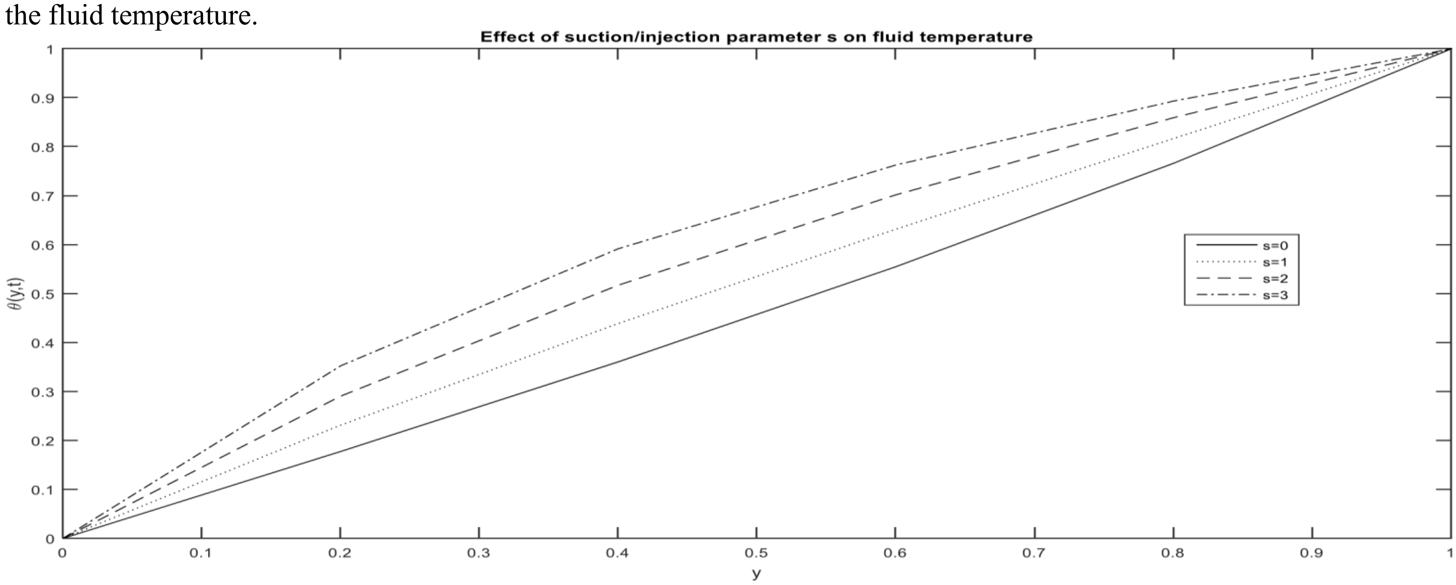

Fig. 8.: Plot of temperature $\theta(y, t)$ against y showing effect s on θ with $\delta = 1$, $Pr = 0.71$, $\omega = 0.5$ *and* $t = 0.01$

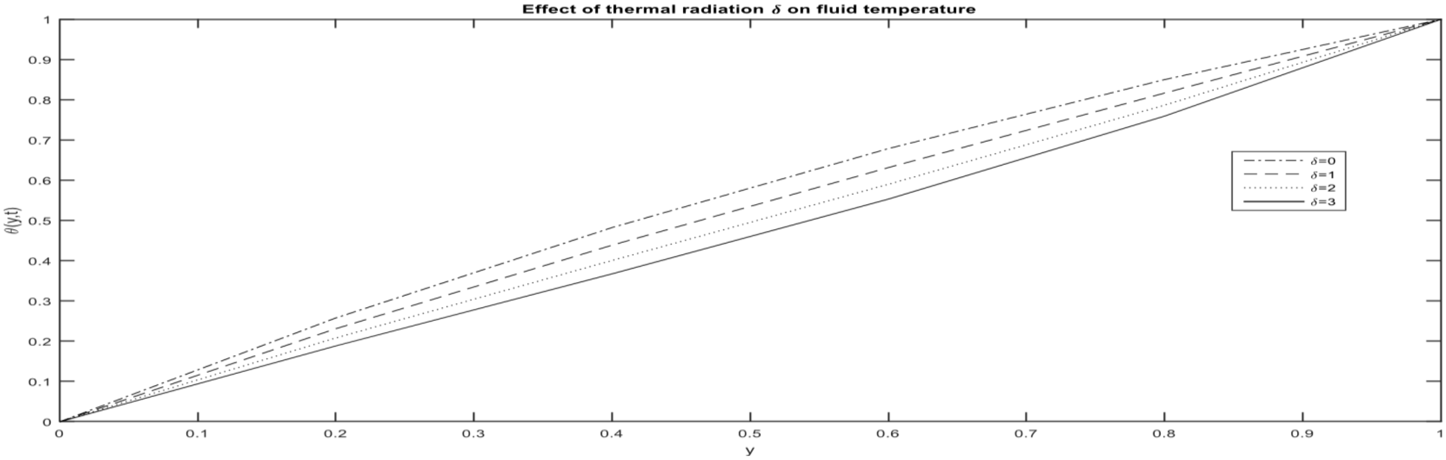

Fig. 9.: Plot of temperature $\theta(y, t)$ against y showing effect of thermal radiation δ with $s = 1$, $Pr = 0.71$, $\omega = 0.5$ and $t = 0.01$

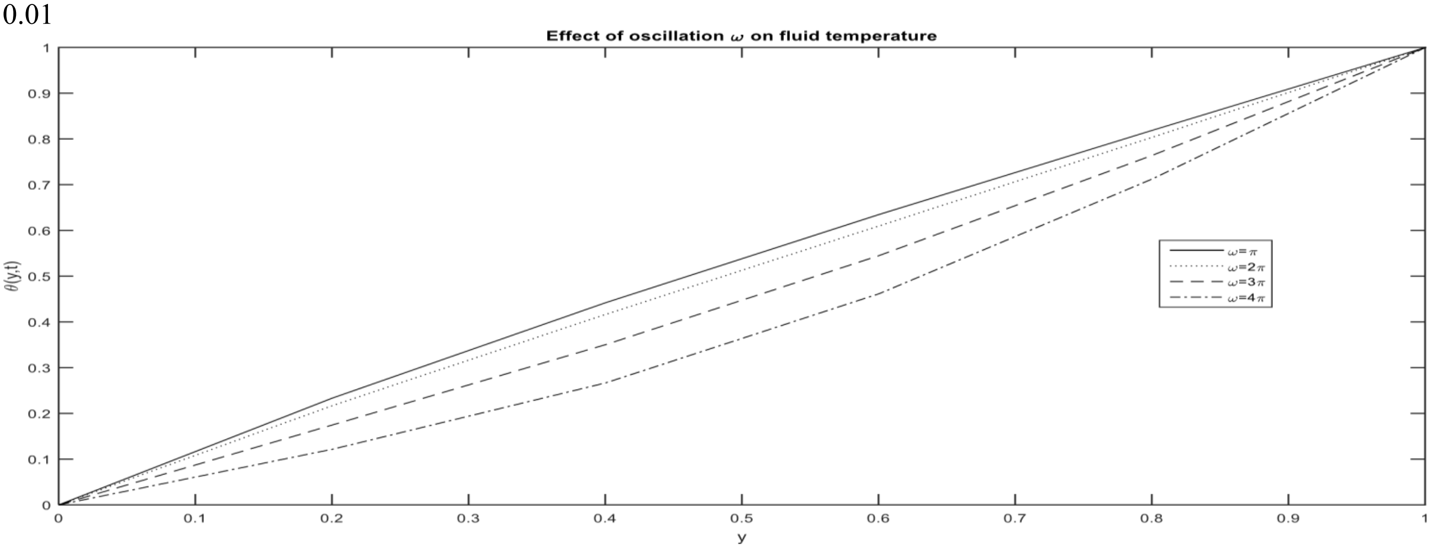

Fig. 10.: Plot of temperature $\theta(y, t)$ against y showing effect of oscillation ω on fluid temperature with $s = 1$, $Pr = 0.71$, $\delta = 1$ and $t = 0.01$

Fig. 11. and **Fig. 12.** illustrate the influence of suction/injection s and frequency of oscillation ω on concentration distribution ϕ. It is evident that increase in both suction/injection parameter s and frequency of oscillation ω increases the concentration distribution ϕ.

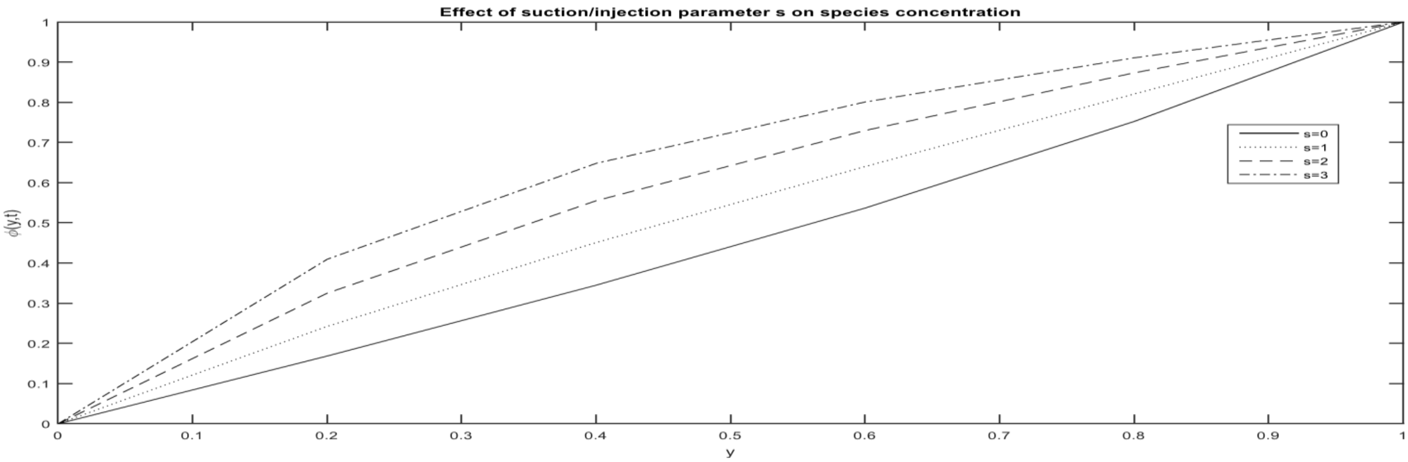

Fig. 11.: Plot of concentration distribution $\phi(y, t)$ against y showing influence s on ϕ with $\omega = \pi$, $Sc = 1$ and $t = 0.01$

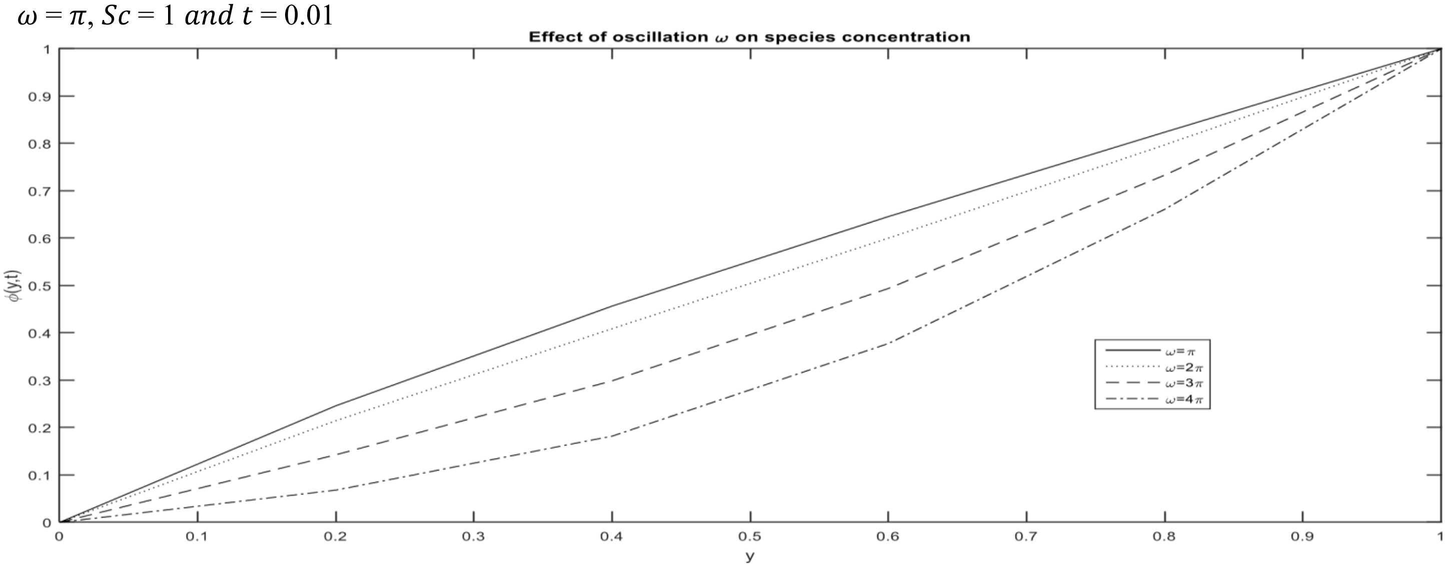

Fig. 12.: Plot of concentration distribution $\phi(y, t)$ against y showing effect of oscillation ω on species concentration with $s = 1$, $Sc = 1$ and $t = 0.01$

5. Conclusion

The effect of heat and mass transfer on unsteady MHD oscillatory flow of fluid in a vertical porous channel has been investigated. The channel is porous and we assumed laminar and incompressible flow. The temperatures prescribed at the channel walls are non – uniform. Magnetic field strength which is uniform is applied transversely to the channel. Closed form solution method is used to solve the dimensionless equations govern the flow and the solutions for velocity, temperature and concentration distribution are obtained. The effects of flow parameters on velocity profile, temperature distribution and species concentration, are presented, discussed and shown graphically in details. It can be concluded that:

(i) Increase in s accelerates the velocity of the fluid and elevates the temperature distribution. (ii) The concentration distribution elevates with higher value of suction and injection parameter.

(iii) Higher value of magnetic parameter diminishes the velocity of the fluid hence, suppresses the turbulence.

(iv) Higher Grashof numbers enhance the velocity of fluid flow.
(v) Higher values of frequency of oscillation enhance the concentration distribution

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