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UNSTEADY MAGNETOHYDRODYNAMIC (MHD) MIXED CONVECTION FLOW OF FLUID IN AN INCLINED CHANNEL WITH HEAT SOURCE

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Abstract: This paper presents the unsteady magnetohydrodynamic (MHD) mixed convection flow of fluid in an inclined channel with heat source. Uniform and asymmetric temperatures are prescribed at the channel walls. A regular perturbation method is employed to solve the momentum and energy equations. The solutions to velocity and temperature profiles are obtained. Computational values of skin friction and Nusselt numbers are obtained and tabulated. Amongst many results, it is found out that the velocity profile declines with increase in magnetic parameter, Reynolds number and Prandtl number. While, it accelerates with increase in Grashof number and heat source parameter. The temperature distribution increases and decreases with increase in heat source parameter and Prandtl number respectively.

Keywords: Mixed convection, MHD, Heat source, Inclined channel, perturbation method

1. Introduction

The study of mixed convection flow in channels has received considerable attention in recent years due to its application in solar collectors, cooling of modern electronic devices and so on. Problems of this nature have been studied by many researchers and excellent reviews on their properties and phenomenon may be found in the literature. For example, Ostrach (1954) studied the problem of combined free and forced convection flow between parallel plates with viscous dissipation in which the wall temperature decreased with height; Tao (1960) studied mixed convection in a vertical parallel – plate channel with uniform wall temperatures. Sparrow et al (1984) observed experimentally flow reversal in pure convection in a one – sided heated vertical channel. The mixed convection in a vertical parallel – plate channel with asymmetric heating, where one plate was heated and the other was adiabatic was studied by Habchi and Acharya (1986). Hadim (1994) analyzed steady laminar forced convection in a channel filled with a fluid-saturated porous medium and containing discrete heat sources on the bottom wall. Hydrodynamic and heat transfer results were reported for two configurations: (1) a fully porous channel, and (2) a partially porous channel, which contains porous layers above the heat sources and is nonporous elsewhere. The flow in the porous medium was modeled using the

Brinkman-Forchheimer extended Darcy model. Heat transfer rates and pressure drop were evaluated for wide ranges of Darcy and Reynolds numbers. Detailed results of the evolution of the hydrodynamic and thermal boundary layers were also provided. The results indicated that as the Darcy number decreases, a significant increase in heat transfer is obtained, especially at the leading edge of each heat source. For fixed Reynolds number, the length-averaged Nusselt number reaches an asymptotic value in the Darcian regime. In the partially porous channel, it was found that when the width of the heat source and the spacing between the porous layers are of the same magnitude as the channel height, the heat transfer enhancement is almost the same as in the fully porous channel while the pressure drop was significantly lower. Barletta (1999) investigated the fully developed and laminar convection in a parallel – plate channel by taking into account both viscous dissipation and buoyancy. Uniform and asymmetric temperatures were prescribed at the channel walls. The velocity field was considered as parallel. A perturbation method was employed to solve the momentum balance equation and energy balance equation. A comparison between the velocity and temperature profiles in the case of laminar forced convection with viscous dissipation was performed in order to point out the effect of buoyancy. The case of convective boundary conditions was also discussed.

Isreal-Cookey et al. (2007) investigated the effects of radiation on combined free and forced convection of a fully developed magnetohydrodynamic (MHD) fluid in a heated infinite vertical channel with viscous dissipation. The temperatures of the channel walls were equal and varied linearly with the heights of the wall. The velocity and the temperature fields together with the flow rate and rate of heat transfer were obtained by invoking the opticslly thin limit for the radiative heat flux in the energy equation. A regular perturbation technique for small Eckert number Ec was employed. They found out that increase in magnetic parameter led to a decrease in the velocity profile. Simultaneous increase in radiation and magnetic parameters led to a progressively flatter velocity. Separate increase in both radiation and magnetic parameters led to decrease in temperature within the channel with a flatter regime recorded for higher values of radiation and magnetic field parameter. Also, the flow rate increased with increase in radiation parameter, whereas the reverse is the case with increase in the magnetic field parameter. The rate of heat transfer decreased with increase in both radiation and magnetic field parameters. Saleh and Hashim (2009) analysed flow reversal phenomena of the fully – developed laminar combined free and forced MHD convection in a vertical plate – channel. The effect of viscous dissipation is taken into account. Idowu et al. (2013) studied the fully developed laminar and MHD convection in a vertical channel with uniform and asymmetric temperatures. The governing equations (momentum and energy balanced equations) has been written in a dimensionless form. A perturbation method has been employed to obtain the solution of the dimensionless velocity u and temperature θ in terms of the perturbation parameter E. It has been pointed out that the effect of magnetic parameter enhances the effect of flow in the downward direction while it lowers this effect in the upward flow. Umamaheswar et al. (2013) dealt with an unsteady magneto hydrodynamic free convective, Viscoelastic, dissipative fluid flow embedded in porous medium bounded by an infinite inclined porous plate in the presence of heat source, and Ohmic heating under the influence of transversely applied magnetic field of uniform strength. The equations governing the fluid flow were solved using a multiple parameter perturbation technique, subject to the relevant boundary conditions. Expressions for velocity and temperature distributions were obtained. Non dimensional skin friction coefficient and the rate of heat transfer in the form of Nusselt number were also derived and illustrated using graphs and tables. The effects of various physical parameters on the above flow quantities were discussed. Ojjela and Kumar (2016) investigated the first-order chemical reaction and Soret and Dufour effects on an incompressible MHD combined free and forced convection heat and mass transfer of a micropolar fluid through a porous medium between two parallel plates. They assumed that there were a periodic injection and suction at the lower and upper plates. The non-uniform temperature and concentration of the plates were assumed to be varying periodically with time. A suitable similarity transformation was used to reduce the governing partial differential equations into nonlinear ordinary differential equations and then solved numerically by the quasilinearization method. The fluid flow and heat and mass transfer characteristics for various parameters were analyzed in detail and shown in the form of graphs. They observed that the concentration of the fluid decreases whereas the temperature of the fluid enhances with the increasing of chemical reaction and Soret and Dufour parameters. Allan and Dardery (2018) considered the unsteady two-dimensional convective heat and mass transfer flow of a viscous, incompressible, electrically conducting optically thin fluid which is bounded by a vertical infinite plane surface. A uniform applied homogeneous magnetic field was considered in the transverse direction with first order chemical reaction. They presented an analytical solution for two-dimensional oscillatory flow on unsteady mixed convection of an incompressible viscous fluid, through a porous medium bounded by an infinite vertical plate in the presence of chemical reaction and thermal radiation. The surface absorbed the fluid with a constant suction and the free stream velocity oscillated about a constant mean value. The resulting nonlinear partial differential equations were transformed into a set of ordinary differential equations using two-term series. They used regular perturbation technique to obtain the solutions for velocity, temperature, concentration, skin friction, Nusselt number, and Sherwood. Mekonnen (2018) studied the problem of unsteady laminar, incompressible, electrically conducting micropolar fluid between inclined channel of rectangular cross-section. It was assumed the flow was under the influence of transverse magnetic field and the walls of the channel have constant temperatures and finite conductivity. The numerical solutions was obtained for the velocity, magnetic field profile, microrotation and temperature fields for various parametric conditions. It was found that velocity, magnetic field profile and micro rotations promote the motion of the fluid with increase the wall conductance. It was also found that both velocity and magnetic field decreases with magnetic parameter increases. Aina and Malgwi (2019) investigated the influence of transverse magnetic field as well as suction/injection on MHD natural convection flow of conducting fluid in an inclined micro-porous-channel. The analytical solutions for velocity profile and temperature profile have been obtained considering the velocity slip and temperature jump conditions at the micro-porous-channel walls. The solution obtained for the velocity has been used to compute the skin friction, while the temperature has been used to compute the Nusselt number. The effect of various flow parameters entering into the problem are discussed with the aid of line graphs. Their results revealed that the impact of inclination angle on fluid velocity is dependent on the value of the wall ambient temperature difference ratio, hence increase in inclination angle yields an enhancement in fluid velocity within the micro-porous-channel for some selected values of the wall ambient temperature difference ratio whereas it displays a dual character for other values. Also, injecting through the micro-porous channel thickens the thermal boundary layer, resulting to weakening the convective current and consequently decreasing the fluid velocity whereas suction weakens the thermal boundary layer yielding an increase in fluid velocity. Khan and Rasheed (2019) studied the magnetohydrodynamic (MHD) mixed convection Maxwell flow of an incompressible nanofluid with magnetic field and heat transfer over a moving plate aligned horizontally. Thermal radiation has also been applied in order to investigate its effects on velocity and temperature variations in the fluid. The Caputo time derivative has been employed to derive the mathematical model. A numerical solution has been obtained using finite difference discretization along with $L1$-algorithm.

Gouda et al (2020) studied the effect of Heat source on an MHD Casson fluid through a vertical fluctuating porous plate. Dimensional non-linear coupled differential equations were transformed into dimensionless similarity variables. The transformed non-dimensional ordinary differential equations of velocity, temperature was solved by Galerkin element technique. The influence of flow parameters like Casson fluid, Permeability, magnetic number, Phase angle, Prandtl number and other physical quantities on velocity along with temperature profiles was explained in detailed via graphs. Skin friction and Nusselt number were also explained through tables. They found out that with increasing the heat source parameter the result in the velocity and temperature increases. Achogo et al. (2021) studied the impacts of velocity, temperature, concentration varieties and magnetic fields on convective intermittent stream on an electrically directing, viscous and incompressible fluid through a permeable medium in a slanted plane. A bunch of coupled partial differential equations emerging from the issue were converted to space dependent ordinary differential equations with a single term perturbation techniques and solved systematically by the technique of undetermined coefficients. They found out that expansion in the Prandtl number declines the temperature, expansion in the Reynolds number reduces the temperature and concentration of the fluid, expansion in the Schmidt number abates the concentration making it more critical at the centre of the flow region, expansion in penetrability prompts expansion in the speed and expansion in the magnetic field prompts decline in the speed, a decrease in the temperature profile is noted owing to the increase in the heat generation, expansion in Dufour number increases both the temperature and speed profiles. Hazoor et al. (2022) dealt with two-dimensional steady boundary layer flow, heat, and mass transfer characteristics of micropolar nanofluid past on exponentially stretching/shrinking surface. They also examined the effect of different physical parameters like magnetic field, buoyancy, thermal radiation, and connective heat transfer. Furthermore, similarity solutions were obtained by similarity transformation on the governing system of partial differential equations. The shooting method with help of the Maple software was used to achieve the numerical solutions of the equations. For the different ranges of the applied parameters, triple solutions were obtained for both cases of the surface. In view of the triple solutions, stability analysis was performed by bvp4c in the MATLAB software, where only first solution was found feasible which was discussed. Their main findings of the first solution indicated that the skin friction, drag force, heat, and mass transfer rates were increasing for the $\lambda > 0$ and decreasing for $\lambda < 0$ as the K is enhanced. The velocity profiles decreased with increase in magnetic, slip velocity, and suction parameters. The temperature profiles increased with increase in magnetic, thermophoresis, thermal radiation, and Brownian motion parameters, whereas concentration profiles reduced with increase in Schmitt number and Brownian motion. Taid and Ahmed (2022) studied the steady two-dimensional MHD free convection flow past an inclined porous plate embedded in the porous medium in the presence of heat source, iSoret effect, and chemical reaction. The non-dimensional

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governing equations were solved by the perturbation technique. The Rosseland approximation was utilized to describe the radiative heat flux in the energy equation. The effect of magnetic parameter, heat source parameter, radiation parameter, Grashofi number, modified Grashofi number, Schmidt number, Prandtl number, porosity parameter, Soreti number, and chemical reaction on velocity, temperature, concentration profiles, skin friction, Nusselt number, and Sherwood number were mainly focussed in discussion with the help of graphs. It was seen that velocity, concentration, and skin friction fall with the increasing value of chemical reaction. Further, temperature, Nusselt number, and Sherwood number increase with the increasing value of chemical reaction. Osman et al. (2022) studied the magnetohydrodynamic (MHD) free convection flow past an infinite inclined plate. In order to explore the effects of velocity, temperature and concentration, Laplace transform technique has been used in this study. They method was adapted to accomplish the analytical solution of governing equations. The impact of various embedded parameter on concentration, velocity and temperature such as chemical reaction, magnetic parameter, radiation and inclination angle has been discussed graphically with numerical results.

This paper presents the unsteady magnetohydrodynamic (MHD) mixed convection flow of fluid in an inclined channel with heat source. Uniform and asymmetric temperatures are prescribed at the channel walls. A regular perturbation method is employed to solve the momentum and energy equations.

2. Problem Formulation

We consider the laminar flow of a Newtonian fluid in the gap between two infinitely-wide plane parallel walls in the presence of magnetic field with heat source. The flow is assumed to be parallel such that U has the only non-vanishing component U along X-axis. The axis orthogonal to the walls of the channel, the gravitational acceleration g and the X-axis lie on the same plane. The flow can is considered to two dimensional i.e. both the velocity field and the temperature field depend only on two spatial conditions. The system under consideration is sketched in Fig.1 where the chosen coordinates axes are (X, Y) are drawn. The wall at $Y = -L$ is kept isothermal with a constant temperature T_1, while the wall at $Y = L$ is subjected to an oscillating temperature. A uniform magnetic field B_0 is applied perpendicular to the inclined channel.

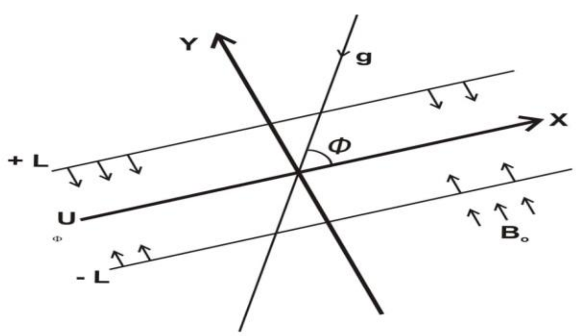

Fig. 1: Schematic configuration of the flow

Using Boussinesq approximation, the momentum and energy equations are given as

$$\rho \frac{\partial U}{\partial t^*} = -\frac{\partial P}{\partial X} + \mu \frac{\partial^2 U}{\partial Y^2} - \sigma B_0^2 U + \rho g \beta \left(T - T_0\right) \cos\phi \tag{1}$$

$$\frac{\partial T}{\partial t^*} = \alpha \frac{\partial^2 T}{\partial Y^2} + \frac{\delta_0 \left(T - T_0\right)}{\Delta T} \tag{2}$$

With the boundary conditions

$$\left.\begin{aligned} &U = 0, T = T_1 \text{ at } Y = -L, t^* \le 0 \\ &U = 0, T = T_2 \text{ at } Y = L, t^* > 0 \end{aligned}\right\} \tag{3}$$

We now introduce the following non-dimensional variables

$$\left.\begin{aligned} &\theta = \frac{T - T_1}{\Delta T}, u = \frac{U}{U_0}, y = \frac{Y}{D}, \lambda = \frac{D^2 P}{\mu U_0}, \Omega = \frac{\omega D^2}{\nu}, P_r = \frac{\nu}{\alpha} \\ &\mathrm{Re} = \frac{U_0 D}{\nu}, G_r = \frac{\beta g \Delta T \cos\phi}{\nu^2}, M^2 = \frac{\sigma B_0^2 U}{\nu}, \delta = \frac{\delta_0 D^2}{\Delta T K}, \Delta T = T_2 - T_1 \end{aligned}\right\} \tag{4}$$

where, $D = 4L$ is the hydraulic diameter.

By employing equation (4), equations (1) – (3) can be written as

$$\Omega \frac{\partial u}{\partial t} = \lambda + \frac{\partial^2 u}{\partial y^2} - M^2 u + \frac{G_r}{\mathrm{Re}} \theta \tag{5}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Omega P_r} \frac{\partial^2 \theta}{\partial y^2} - \frac{\delta}{\Omega P_r} \theta \tag{6}$$

And the dimensionless boundary conditions

$$\left.\begin{aligned} &u = 0, \theta = 0 \text{ at y} = -\frac{1}{4}, t \le 0 \\ &u = 0, \theta = 1 \text{ at y} = \frac{1}{4}, t > 0 \end{aligned}\right\} \tag{7}$$

3. Method/Solution of the Problem

In this section, we employed the regular perturbation method to solve equations (5) and (6) subject to the boundary conditions in equation (7). This is done by expanding the functions $u(y,t)$ and $\theta(y,t)$ as

$$\left.\begin{aligned} &u(y,t) = u_0(y) + \varepsilon u_1(y) e^{i\omega t} + O\left(\varepsilon^2\right) \\ &\theta(y,t) = \theta_0(y) + \varepsilon \theta_1(y) e^{i\omega t} + O\left(\varepsilon^2\right) \end{aligned}\right\} \tag{8}$$

and we choose $\lambda = -\lambda_0 e^{i\omega t}$ as the prescribed pressure gradient and the amplitude of oscillation $\varepsilon << 1$.

In view of equation (8), the periodic and non – periodic terms of the equations (5) and (6) are

$$\frac{\partial^2 u_0}{\partial y^2} - M^2 u_0 = -\lambda_0 - E\theta_0 \tag{9}$$

$$\frac{\partial^2 u_1}{\partial y^2} - \left(M^2 + i\omega\Omega\right) u_1 = -E\theta_1 \tag{10}$$

$$\frac{\partial^2 \theta_0}{\partial y^2} - \delta\theta_0 = 0 \tag{11}$$

$$\frac{\partial^2 \theta_1}{\partial y^2} - \left(\delta + i\omega\Omega\right)\theta_1 = 0 \tag{12}$$

And the boundary conditions are

$$\left.\begin{aligned} &u_0 = 0, u_1 = 0, \theta_0 = 0, \theta_1 = 0 \text{ at } y = -\frac{1}{4} \\ &u_0 = 0, u_1 = 0, \theta_0 = 1, \theta_1 = 0 \text{ at } y = \frac{1}{4} \end{aligned}\right\} \tag{13}$$

Since, equations (9) and (10) are coupled differential equations, we first solved for equations (11) and (12). Therefore, we obtain the following

$$\theta_0(y) = c_1 e^{\sqrt{\delta}y} + c_2 e^{-\sqrt{\delta}y} \tag{14}$$

$$\theta_1(y) = c_3 e^{\sqrt{A}y} + c_4 e^{-\sqrt{A}y} \tag{15}$$

$$u_0(y) = c_5 e^{My} + c_6 e^{-My} + K_0 + K_1 e^{\sqrt{\delta}y} + K_2 e^{-\sqrt{\delta}y} \tag{16}$$

$$u_1(y) = c_7 e^{\sqrt{B}y} + c_8 e^{-\sqrt{B}y} + K_3 e^{\sqrt{A}y} + K_4 e^{-\sqrt{A}y} \tag{17}$$

Where,

$$c_1=\frac{1}{e^{\frac{\sqrt{\delta}}{4}}-e^{-\frac{\sqrt{\delta}}{4}}},c_2=c_1e^{\frac{\sqrt{\delta}}{4}},A=\delta+i\omega\Omega,c_3=\frac{\sin m\pi}{e^{\frac{-\sqrt{A}}{4}}-e^{\frac{3\sqrt{A}}{4}}},c_4=\frac{\sin m\pi}{e^{\frac{-3\sqrt{A}}{4}}-e^{\frac{\sqrt{A}}{4}}},E=\frac{G_r}{\mathrm{Re}}$$

$$K_0=\frac{\lambda_0}{M^2},K_1=\frac{-Ec_1}{\delta-M^2},K_2=\frac{-Ec_2}{\delta-M^2},A_1=K_0+K_1e^{\frac{-\sqrt{\delta}}{4}}+K_2e^{\frac{\sqrt{\delta}}{4}},$$

$$A_2=K_0+K_1e^{\frac{\sqrt{\delta}}{4}}+K_2e^{\frac{-\sqrt{\delta}}{4}},c_5=\frac{A_2e^{\frac{M}{4}}-A_1e^{-\frac{M}{4}}}{e^{-\frac{M}{2}}-e^{\frac{M}{2}}},c_6=A_2e^{\frac{M}{4}}-c_5e^{\frac{M}{2}},B=M^2+i\omega\Omega, \tag{18}$$

$$K_3=\frac{-Ec_3}{A-B},K_4=\frac{-Ec_4}{A-B},A_3=K_3e^{\frac{-\sqrt{A}}{4}}+K_4e^{\frac{\sqrt{A}}{4}},A_4=K_3e^{\frac{\sqrt{A}}{4}}+K_4e^{\frac{-\sqrt{A}}{4}},$$

$$c_7=\frac{A_4e^{\frac{\sqrt{B}}{4}}-A_3e^{-\frac{\sqrt{B}}{4}}}{e^{-\frac{\sqrt{B}}{2}}-e^{\frac{\sqrt{B}}{2}}},c_8=-A_4e^{\frac{\sqrt{B}}{4}}-c_7e^{\frac{\sqrt{B}}{2}}$$

In view of equations (8), (14), (15), (16) and (17), the solutions to velocity and temperature profiles are respectively,

$$\begin{aligned}u(y,t)&=c_5e^{My}+c_6e^{-My}+K_0+K_1e^{\sqrt{\delta}y}+K_2e^{-\sqrt{\delta}y}+\\&\varepsilon\left(c_7e^{\sqrt{B}y}+c_8e^{-\sqrt{B}y}+K_3e^{\sqrt{A}y}+K_4e^{-\sqrt{A}y}\right)e^{i\omega t}\end{aligned} \tag{19}$$

$$\theta(y,t)=c_1e^{\sqrt{\delta}y}+c_2e^{-\sqrt{\delta}y}+\varepsilon\left(c_3e^{\sqrt{A}y}+c_4e^{-\sqrt{A}y}\right)e^{i\omega t} \tag{20}$$

3.2 The Skin Friction τ and Nusselt number N_u

The skin frictions at the walls $y=-L$ and $y=L$ are respectively given as

$$\tau_1=\frac{2}{\mathrm{Re}}\frac{\partial u}{\partial y}\bigg|_{y=\frac{-1}{4}}=\frac{2}{\mathrm{Re}}\left[\begin{aligned}&Mc_5e^{\frac{-M}{4}}-Mc_6e^{\frac{M}{4}}+\sqrt{\delta}K_1e^{\frac{-\sqrt{\delta}}{4}}-\sqrt{\delta}K_2e^{\frac{\sqrt{\delta}}{4}}+\\&\varepsilon\left(\sqrt{B}c_7e^{\frac{-\sqrt{B}}{4}}-\sqrt{B}c_8e^{\frac{\sqrt{B}}{4}}+\sqrt{A}K_3e^{\frac{-\sqrt{A}}{4}}-\sqrt{A}K_2e^{\frac{\sqrt{A}}{4}}\right)e^{i\omega t}\end{aligned}\right] \tag{21}$$

$$\tau_2=\frac{2}{\mathrm{Re}}\frac{\partial u}{\partial y}\bigg|_{y=\frac{1}{4}}=\frac{2}{\mathrm{Re}}\left[\begin{aligned}&Mc_5e^{\frac{M}{4}}-Mc_6e^{\frac{-M}{4}}+\sqrt{\delta}K_1e^{\frac{\sqrt{\delta}}{4}}-\sqrt{\delta}K_2e^{\frac{-\sqrt{\delta}}{4}}+\\&\varepsilon\left(\sqrt{B}c_7e^{\frac{\sqrt{B}}{4}}-\sqrt{B}c_8e^{\frac{-\sqrt{B}}{4}}+\sqrt{A}K_3e^{\frac{\sqrt{A}}{4}}-\sqrt{A}K_2e^{\frac{-\sqrt{A}}{4}}\right)e^{i\omega t}\end{aligned}\right] \tag{22}$$

The Nusselt numbers at the walls $y=-L$ and $y=L$ are respectively given as

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$$Nu_1 = \left.\frac{\partial\theta}{\partial y}\right|_{y=\frac{-1}{4}} = \left[\sqrt{\delta}c_1 e^{\frac{-\sqrt{\delta}}{4}} - \sqrt{\delta}c_2 e^{\frac{\sqrt{\delta}}{4}} + \varepsilon\left(\sqrt{A}c_3 e^{\frac{-\sqrt{A}}{4}} - \sqrt{A}c_4 e^{\frac{\sqrt{A}}{4}}\right)e^{i\omega t}\right] \quad (23)$$

$$Nu_2 = \left.\frac{\partial\theta}{\partial y}\right|_{y=\frac{1}{4}} = \left[\sqrt{\delta}c_1 e^{\frac{\sqrt{\delta}}{4}} - \sqrt{\delta}c_2 e^{\frac{-\sqrt{\delta}}{4}} + \varepsilon\left(\sqrt{A}c_3 e^{\frac{\sqrt{A}}{4}} - \sqrt{A}c_4 e^{\frac{-\sqrt{A}}{4}}\right)e^{i\omega t}\right] \quad (24)$$

4. Results

The unsteady magnetohydrodynamic (MHD) mixed convection flow of fluid in an inclined channel with heat source has been analyzed. The impact of magnetic parameter M, Reynolds number Re, Grashof number G_r, Prandtl number P_r and heat source parameter δ are plotted graphically on different flow fields. The default values for the governing flow parameters are taken as

$\lambda = 0.30, \Omega = 0.20, \omega = 0.05, t = 0.05, \ \mathrm{Re} = 0.05,$

$G_r = 5, P_r = 0.71, M = 0.30, \delta = 0.05, \varepsilon = 0.005.$

Tables 1, 2 and 3 presented the calculated values of skin friction (flow rate) and Nusselt number at the lower ($y = -L$) and upper walls ($y = L$) respectively for the different variations in the governing flow parameters.

Table 1. The skin frictions τ_1 and τ_2 at the lower and upper wall respectively with variation in M, Re and G_r

M	τ_1	τ_2	Re	τ_1	τ_2	G_r	τ_1	τ_2
1.00	1.1420	−1.8700	1.00	1.1420	−1.8700	5.00	1.1420	−1.8700
2.00	1.0603	−1.7794	2.00	0.3941	−0.5761	10.00	1.8496	−3.3056
3.00	0.0603	−1.6523	3.00	0.2234	−0.3043	15.00	2.5571	−4.7412
4.00	0.8246	−1.5111	4.00	0.1528	−0.1983	20.00	3.2647	−6.1769

Table 2. The skin frictions τ_1 and τ_2 at the lower and upper wall respectively with variation in P_r and δ

P_r	τ_1	τ_2	δ	τ_1	τ_2
0.11	1.1420	−1.8700	1.00	1.1420	−1.8700
0.31	1.1429	−1.8711	2.00	1.1222	−1.8472
0.51	1.1439	−1.8722	3.00	1.1033	−1.8254
0.71	1.1448	−1.8733	4.00	1.0854	−1.8045

Table 2. The Nusselt numberss Nu_1 and Nu_2 at the lower and upper wall respectively with variation in P_r and δ

P_r	Nu_1	Nu_2	δ	Nu_1	Nu_2
0.11	1.7099	1.8453	1.00	1.6882	1.8906
0.31	1.6414	1.9863	2.00	1.6956	1.8753
0.51	1.5766	2.1229	3.00	1.7028	1.8602
0.71	1.5154	2.2554	4.00	1.7099	1.8453

Table 1 shows the computational values of skin frictions τ_1 and τ_2 at the lower and upper wall respectively for flow parameters M, Re and G_r. It can be seen that the skin friction at the lower wall plunge with rising data of M because the magnetic field tends to decelerate the fluid flows and hence the surface declines. But the magnetic of the skin friction at the upper wall increases. Increase in Reynolds number Re decreases and increases the skin friction at the lower and upper walls respectively. Higher values of Grashof number G_r respectively increases and decreases the skin friction at the lower and upper walls.

The computational values of skin friction τ_1 and τ_2 at the lower and upper wall respectively for flow parameters P_r and δ are presented in Table 2. It can be deduced that the skin friction decreases and increases at the lower and upper walls respectively with higher values of Prandtl number P_r. But increase in heat source parameter δ decreases and increases the magnitude of skin friction at the lower and upper walls respectively.

Table 3 illustrates the computational values of Nusselt numbers Nu_1 and Nu_2 at the lower and upper wall respectively with variation in P_r and δ. It can be seen that the rate of heat transfer at the lower wall declines while it enhances at the upper wall with the increase in Prandtl number P_r. Higher values of source term parameter δ increases and decreases the magnitude of Nusselt number at the lower and upper walls respectively.

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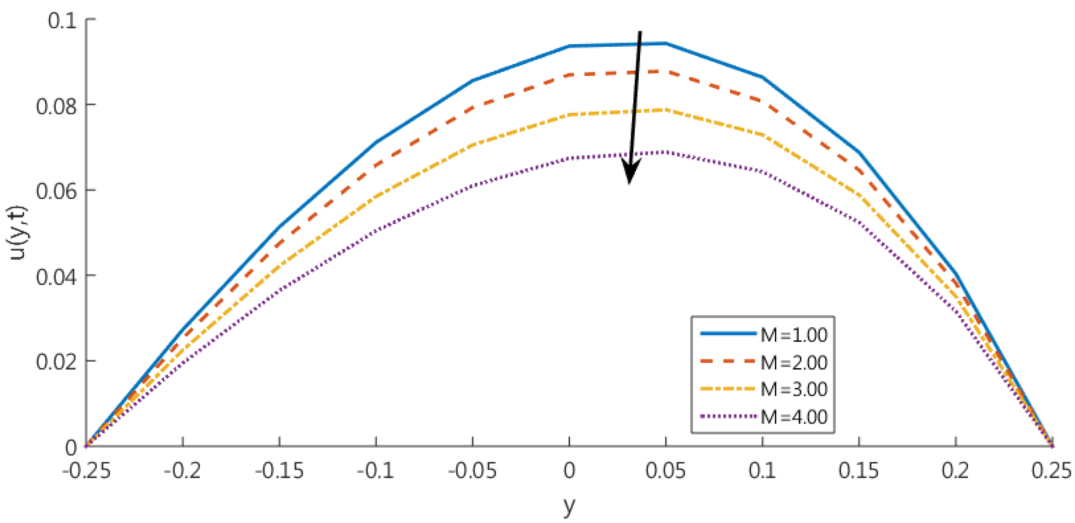

Fig. 2: Influence of Magnetic parameter M on Velocity profile u

Fig. 2 depicts the influence of magnetic parameter M on velocity profile. It can be seen that increase in magnetic parameter decrease the velocity. Generally, when magnetic field is applied to any fluid, the apparent viscosity of the fluid increases to the point of becoming viscous elastic solid and there decreases the flow of the fluid. It is of great interest that the yield stress of the fluid can be controlled accurately through the variation of the magnetic field. Basically, the role of the magnetic field is to suppress turbulence.

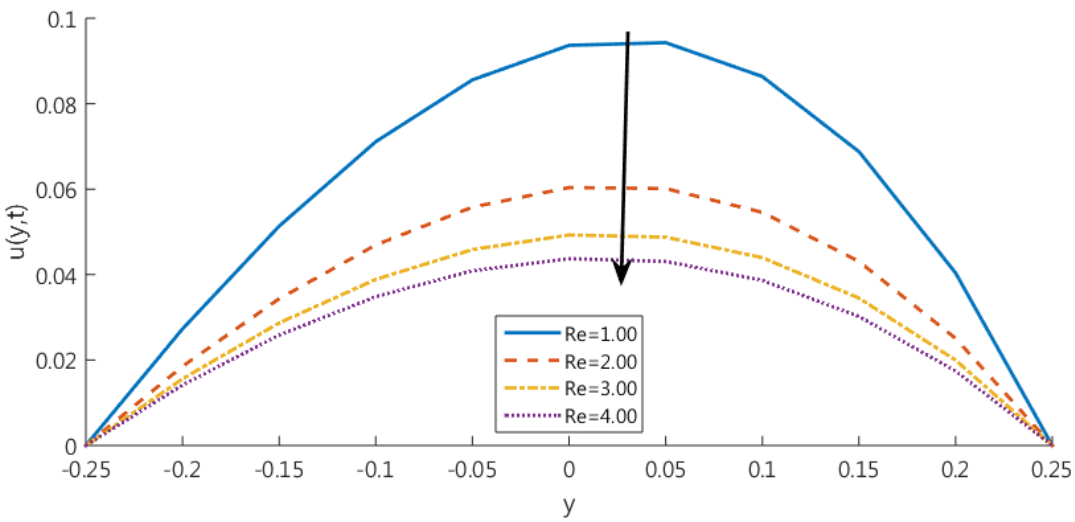

Fig. 3: Influence of Reynolds number Re Velocity profile u

The influence of Reynolds number Re is illustrated in fig. 3. It is observed that the velocity decreases with increase in Reynolds number. Generally, when Reynolds number is

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high the flow is turbulent this is due to increase in velocity. When the Reynolds number is low, the flow of fluid is larminar.

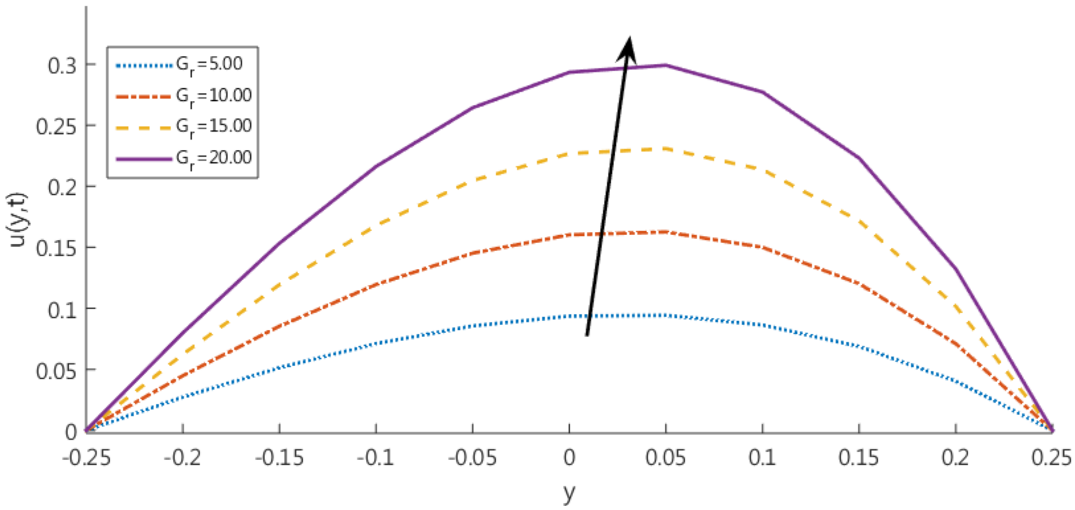

Fig. 4: Influence of Grashof number G_r Velocity profile u

Fig.4 depicts the influence of Grashof number G_r on velocity profile. It is seen that the velocity field increases significantly with increase in Grashof number. To this effect, at higher Grashof number, the flow at the boundary is turbulent, while at lower Grashof number, the flow at the boundary is laminar.

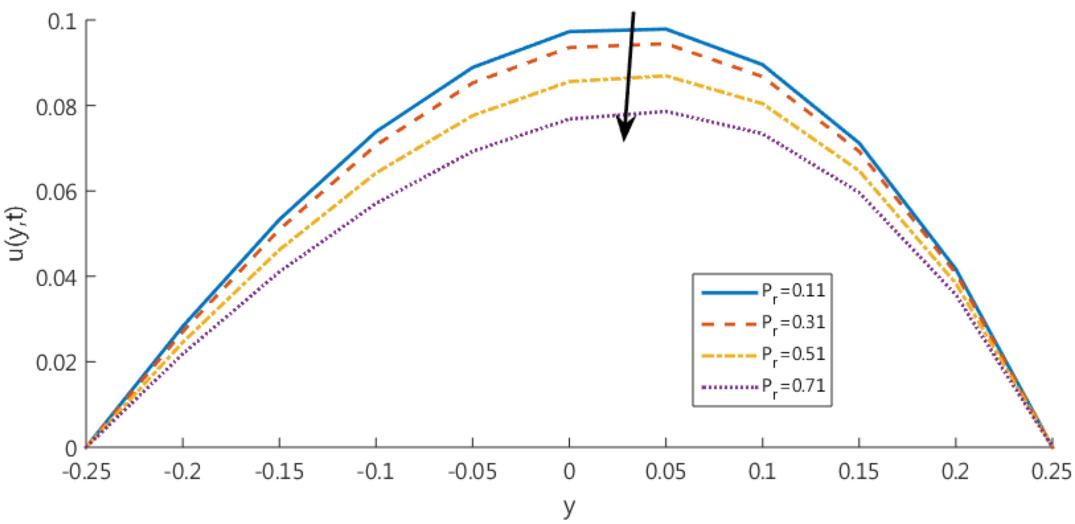

Fig. 5: Influence of Prandtl number P_r Velocity profile u

Fig. 5 is plotted to show the influence of Prandtl number P_r on velocity profile. The parameter P_r is the proportion of kinematic viscosity and thermal diffusivity which changes physically with temperature. However, increase in P_r depict the domination of momentum diffusivity. Since, it is inversely proportional to thermal diffusivity, increase in Prandtl number led to the decrease in velocity profile.

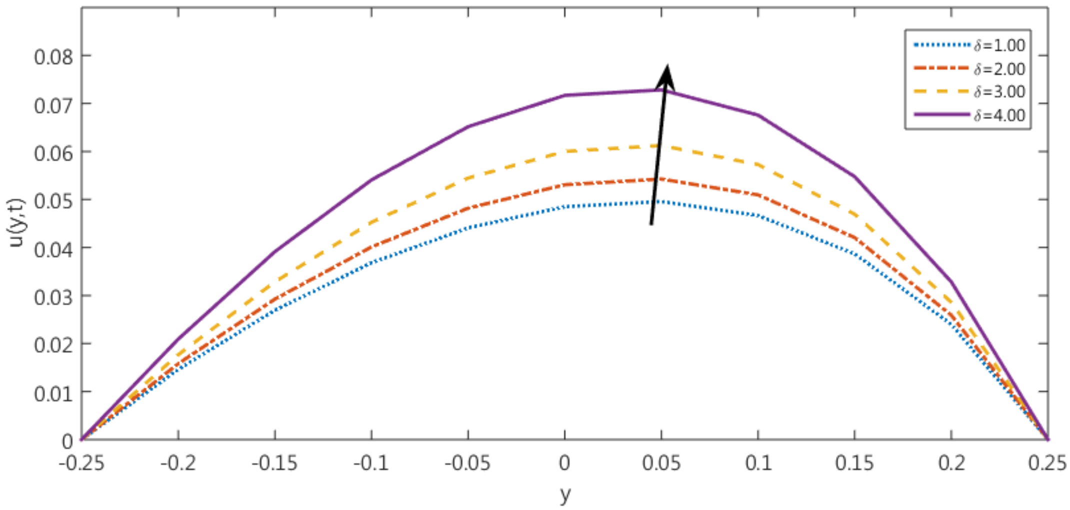

Fig. 6: Influence of Radiation parameter δ on Velocity profile u

Fig.6 demonstrates the influence of heat source parameter δ on velocity profile. It is seen that the velocity profile increases significantly with increase in heat source parameter δ. The heat source parameter is the conversion of thermal energy which generates the thermal motion of particles in matter. This could be attributed due to thermal excitation.

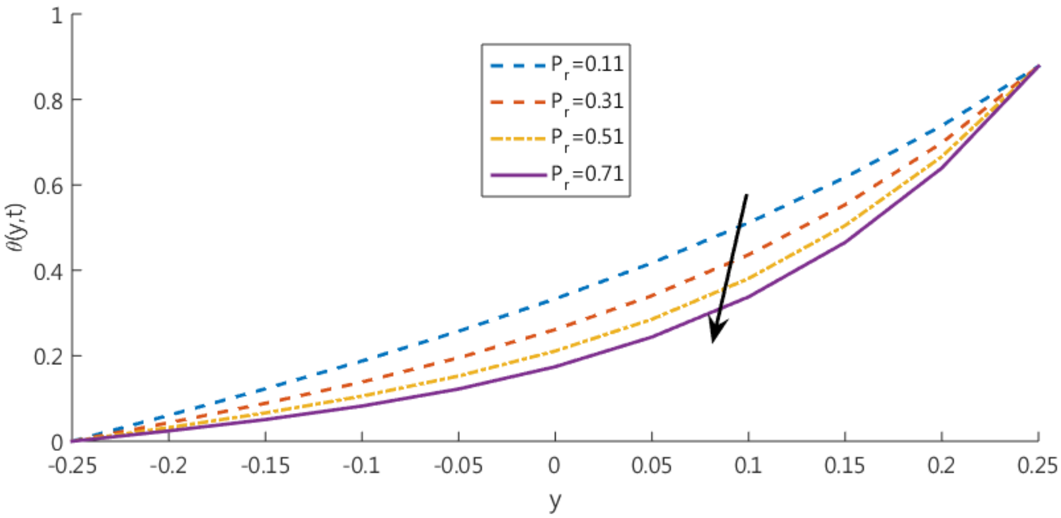

Fig. 7: Influence of Prandtl number P_r on Temperature distribution θ

The influence of Prandtl number P_r on velocity profile is illustrated in fig. 7. Prandtl number is the proportion of kinematic viscosity and thermal diffusivity which changes physically with temperature. For example, water $P_r = 7.0$ (at 20^0C) and Ammonia $P_r = 1.38$ decline more rapidly than air $P_r = 0.71$. However, increase in P_r depict the domination of thermal and momentum diffusivity respectively. Prandtl number is used to determine whether heat transport occurs with either conduction or convection process. Since, Prandtl number is inversely proportional to thermal diffusivity so that increasing P_r led to the decrease in temperature profile.

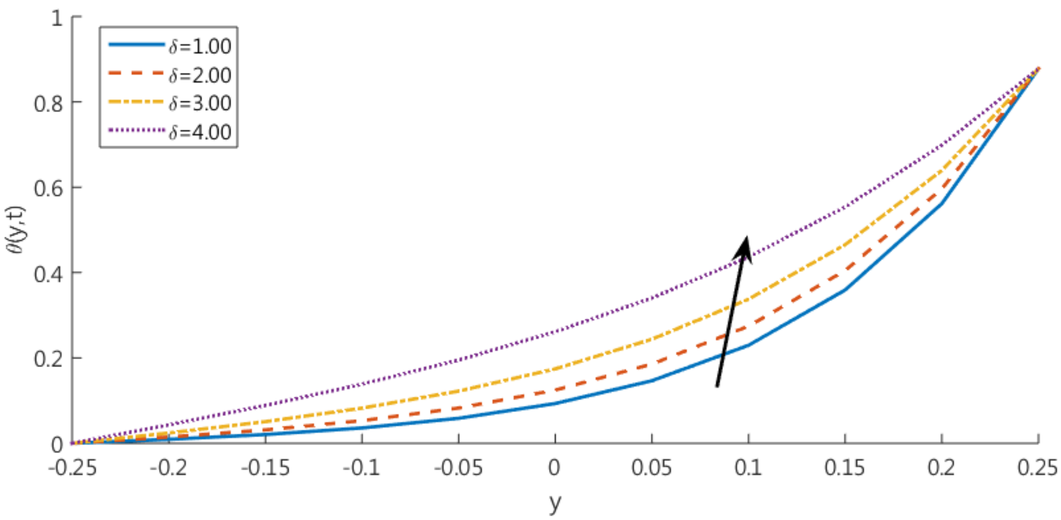

Fig. 8: Influence of Heat source parameter δ on Temperature distribution θ

Fig. 8 depicts the temperature field for increment of heat source parameter δ. Here, the heat source is known as electromagnetic radiation or the conversion of thermal energy which generates the thermal motion of particles in matter. This could be attributed due to thermal excitation. The temperature field is affected significantly with increase in heat source parameter δ. The heat source parameter for a medium which contains it inevitably has pressure and density gradients, and the treatment requires the use of hydrodynamics.

5. Conclusion

The unsteady magnetohydrodynamic (MHD) mixed convection flow of fluid in an inclined channel with heat source has been studied. Uniform and asymmetric temperatures are prescribed at the channel walls. A regular perturbation method is employed to solve the momentum and energy equations.

The main findings are summarized below;

(i) Velocity profile declines with higher values of magnetic parameter, Reynolds number and Prandtl number.

(ii) Upsurging data of Grashof number and heat source parameter accelerate the fluid velocity.

(iii) Temperature distribution profile decreases with increase in Prandtl number.

(iv) Large values of heat source parameter enhances the temperature distribution significantly.

(v) Increment of magnetic parameter, Reynolds number and heat source parameter respectively decrease and increase the magnitude of the skin friction at lower and upper walls.

(vi) Increment of Grashof number and Prandtl number increase and decrease the skin friction at the lower and upper walls respectively.

(vii) Higher values of Prandtl number decreases and increases the rate of heat transfer at the lower and upper walls respectively.

(viii) The rate of heat transfer increases and decreases with the increment of heat source parameter respectively at the lower and upper walls.

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